

## VARIATIONS IN TSUNAMI-ATTENUATION ABILITY OF COASTAL FORESTS WITH AGE BASED ON A FOREST GROWTH MODEL

Toshiyuki Asano<sup>1</sup>, Haruki Omoya<sup>2</sup>, and Yoshinaga Setoguchi<sup>3</sup>

### Abstract

Recently, coastal forests have received considerable attention because they provide concurrent benefits regarding tsunami-disaster mitigation and coastal-environment preservation. An important consideration concerning the use of coastal forests as countermeasures against tsunamis is that the properties of individual trees and of the entire forest stand vary with age. Consequently, the tsunami-mitigation performance of coastal forests is time-dependent and it reflects changes of topography and stand density. Depending on the stage of tree growth, not only the trunk but also the crown might contribute flow resistance. As coastal forests often grow in fields on sandy dunes within a certain distance of the shoreline, such topographic effects on tsunami run-up should also be considered. The objective of this study was to assess the tsunami-attenuation ability of coastal forests by considering the temporal developments of trees and forest stands, and by incorporating realistic beach topographies.

**Key words:** coastal forests, forest ecology, disaster mitigation, time-varying performance, tsunami attenuation, tsunami run-up

### 1. Introduction

After the 2011 off the Pacific coast of Tohoku tsunami, the tsunami-attenuation ability of coastal forests has been examined. In this event, some coastal forests over a vast area were destroyed by the huge hydrodynamic force of the tsunami. However, some forests that survived demonstrated mitigating effects against the tsunami-induced flows because of their drag resistance and by blocking drifting material. Okada et al. (2012) conducted house-destruction surveys for areas both protected and unprotected by coastal forests, where other conditions such as ground level and tsunami height were the same. Their findings confirmed distinct mitigation effects of coastal forests. Furthermore, following the great off-Sumatra tsunami in 2004, researchers found that areas landward of dense mangroves were damaged significantly less than other areas (Danielson et al., 2005).

To utilize coastal forests for the protection of hinterland from a disastrous tsunami, a forest of sufficient width and density is needed. However, if a coastal forest alone is considered inadequate for protection against tsunamis, it could be supplemented by other countermeasure structures such as dikes, dunes, and embankments (an integrated tsunami defense system is illustrated in Fig. 1). A committee for the “Restoration of coastal disaster mitigation forests in the great Tohoku earthquake,” organized by the Japan Forestry Agency, has identified coastal forests as an important element of tsunami defense systems. Actually, a tree-planting project named “Restoration of Green Knot” (Fig. 2) has been implemented in some tsunami devastated areas. These trees will grow and develop into a forest; however, it is vital that future forests exhibit greater resistance against tsunami force for optimal mitigation effect.

For the practical application of forests as a tsunami-mitigation measure, the biological and ecological aspects of trees/forests should be considered. All the characteristics of trees/forests, such as height, trunk diameter, and stand density, are dependent on their stage of growth; thus, their tsunami-attenuation ability has temporal variability. One of the authors investigated the time-varying tsunami attenuation ability of coastal forests with stand age by applying knowledge of forest ecology and tree morphology (Asano, 2008a).

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<sup>1</sup> Dept. Ocean Civil Eng., Kagoshima Univ., 1-14-7 Korimoto, Kagoshima, JAPAN, [asano@oce.kagoshima.ac.jp](mailto:asano@oce.kagoshima.ac.jp)

<sup>2</sup> Wakayama Prefecture, 1-1 Komatsubara, Wakayama, JAPAN, [omoya\\_h0001@pref.wakayama.lg.jp](mailto:omoya_h0001@pref.wakayama.lg.jp)

<sup>3</sup> Yachiyo Engineering Co. Ltd., Nishi-Ochiai, 2-18-12, Shinjuku Tokyo, JAPAN, [ys-setoguchi@yachiyo-eng.co.jp](mailto:ys-setoguchi@yachiyo-eng.co.jp)

This study proposed a conceptual model for the time-varying tsunami-attenuation ability of coastal forests, which incorporated several simplifications: only the trunks of coastal trees were considered to generate fluid resistance, tsunami waves passing through the forests were described by a linear wave model, and the topography on which the forest stood was assumed horizontal.

The objective of the present study was to assess the tsunami-reduction ability of coastal forests by incorporating realistic field conditions. As factors generating fluid resistance against a tsunami, not only the trunk but also the crown (comprising branches and leaves) of the coastal trees were considered. The growths of these parts of trees were modeled based on knowledge of forest ecology, and the proportionalities of such elements were modeled with reference to tree morphology. As coastal forests often grow on sandy dunes within a certain distance of the shoreline, such topographic effects on tsunami run-up were also investigated. To represent the transformation of a tsunami wave during its passage through a coastal forest on a beach, the nonlinear shallow water equation was adopted.

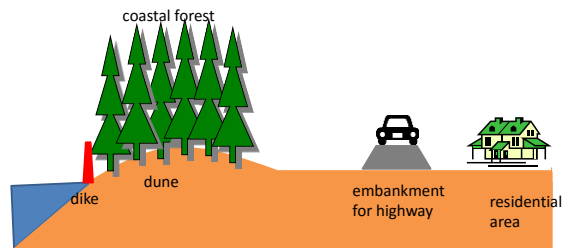


Fig. 1 Integrated coastal defense system



Fig. 2 Tree-planting project (Arahama, Sendai)

## 2. Forest growth model under self-thinning

Initially, young trees planted as shown in Fig. 2 will grow individually. Competition among the trees will begin when their crowns contact each other. If a particular tree grows at a slower rate than surrounding trees, it will receive less sunlight, and the inferior environmental conditions will be exacerbated, leading to self-thinning (Fig. 3). In forest science, a unit of a forest in which all trees within the area are regarded homogeneous in terms of species and age, is called a “forest stand” (Vanclay, 1994). The growth of a forest stand has certain limitations due to competition (Hozumi, 1977).

Here, we adopted the model of Minowa (1982) for the evolution of stand density, i.e., the number of trees per unit area,  $N$ . Generally, the temporal change of the number of vegetation (also for animal) communities under environmental restriction can be expressed as

$$\frac{dN}{dt} = k(N_* - N), \quad (1)$$

where  $N_*$  is the upper limit value and  $k$  is the rate (positive for decay, negative for growth). The logarithmic transformation  $N \rightarrow \ln N$ ,  $N_* \rightarrow \ln N_*$  is introduced in Eq. (1), and the solution of the transformed equation can be expressed by the following Gompertz-type growth equation:

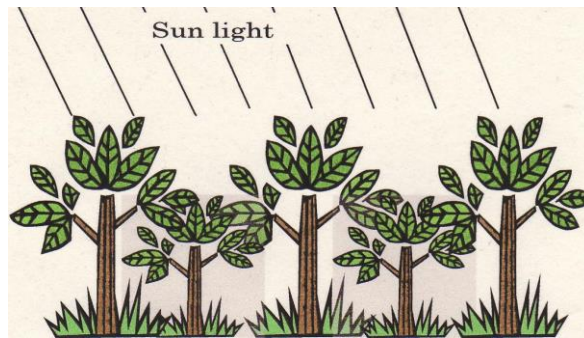


Fig. 3 Schematic self-thinning process generating superior and inferior trees

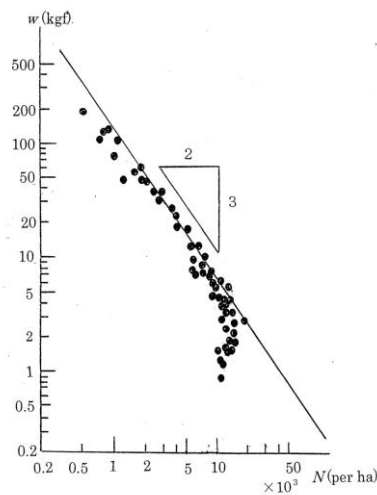


Fig. 4 Stand density  $N$  vs. average individual tree weight  $w$  (Yoda, 1971)

$$(N/N_*) = (N_0/N_*) \exp(-k(t-t_0)), \quad (2)$$

where  $N_0$  and  $t_0$  denote the initial stand density and initial time, respectively. The Gompertz-type growth curve is able to express both increasing and decreasing temporal variation. Khilmi (1967) found that the decreasing property of stand density  $N$ , after canopy closure, could be expressed by Eq. (2).

The characteristics of the elements of individual trees, such as height  $h_{tree}$  and trunk diameter  $d_o$ , are also constrained by stand density  $N$ . Yoda (1971) proposed the following 3/2 power law for the relationship between average individual tree weight  $w$  and maximum density  $N_*$  under a self-thinning situation:

$$wN_*^{3/2} = \text{Const.} \quad (3)$$

The above equation describes the full density condition illustrated by the line in Fig. 4. Minowa (1982) proposed a mathematical model to describe the time-dependent trajectory between  $w$  and  $N$ , before this condition is reached, based on Eqs. (1) and (2). He applied his theory to an idealized forest to satisfy the assumptions of the model, i.e., a pure forest of uniform age without artificial thinning or logging. Figure 5 shows the results of a field survey for white pine forest to examine the validity of his theory (Spurr et al., 1957).

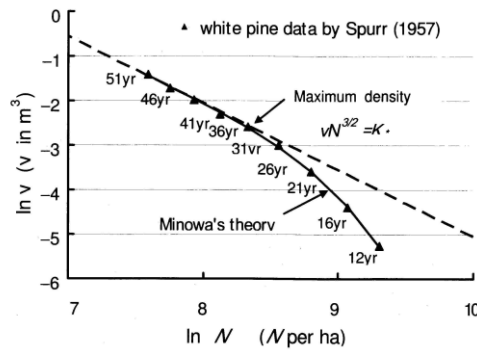


Fig. 5 Time trajectory between stand density  $N$  vs. average individual tree weight  $w$  based on Minowa's model

The agreement between the predicted and measured data is surprisingly good, even though several parameters such as  $k$  and  $N^*$  are tuned. Thus, we adopted Minowa's time evolution model for stand density  $N$  (for details, see Asano (2008a)).

### 3. Model of temporal growth for each element of a coastal tree

As the specific densities of trees are considered similar, Eq. (3) should remain valid if tree weight  $w$  were replaced by tree volume  $v$ , which can be evaluated approximately based on a cylinder:

$$v \propto d_0^3 \propto h_{tree}^3. \quad (4)$$

According to tree morphology, each element of a tree possesses certain proportionality (allometry) depending

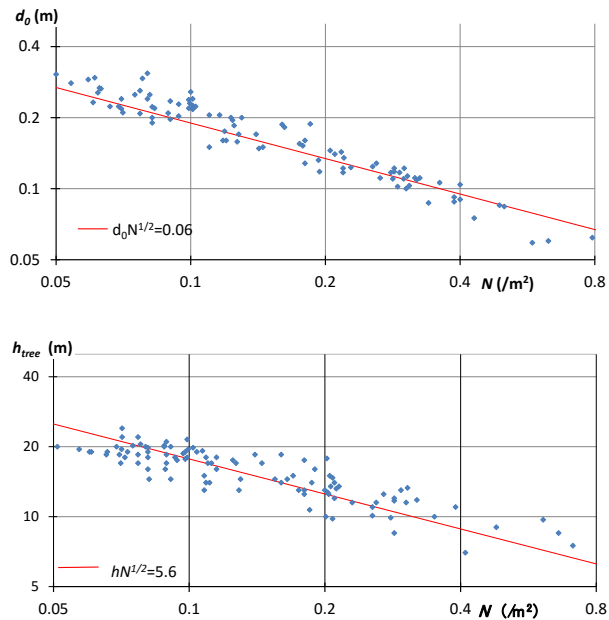


Fig. 6 Average trunk diameter  $d_0$  and average tree height  $h_{tree}$  vs. stand density  $N$  (re-analysis on data measured by Kira and Yoda (1957))

on the tree species(Asano, 1988b). Thus, the following relations can be deduced from Eqs. (3) and (4):

$$d_0 N^{1/2} = C_1, \quad h_{tree} N^{1/2} = C_2 \quad (5)$$

where  $C_1$  and  $C_2$  are constants.

Yoda (1971) confirmed the relationship of Eq. (5) based on their field measurements of a natural red pine forest (Fig. 6). Based on a regression analysis of these data, we adopted the constant values of  $C_1 = 0.06$  and  $C_2 = 5.6$ .

The temporal evolution of tree weight  $w$  can be evaluated based on the time variation of stand density  $N$  following Minowa's model. Therefore, reflecting the temporal variation of  $w$ , the trunk diameter  $d_0$ , tree height  $h_{tree}$ , branch height  $h_{br}$ , and crown width  $w_{crown}$  can also be described as a function of forest age  $\tau$ . Figure 7 illustrates the temporal variation of the stand density  $N$  and trunk diameter at breast height  $d_0$ .

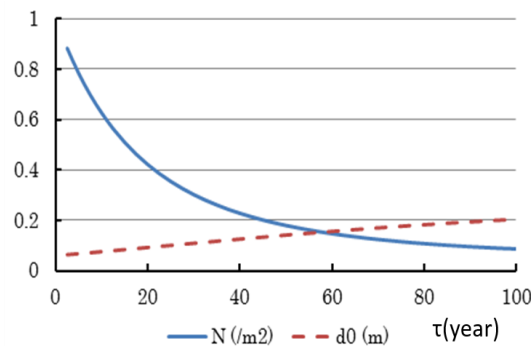


Fig. 7 Temporal variation of stand density  $N$  and trunk diameter at breast height  $d_0$

#### 4. Numerical analysis of tsunami flow over tree-covered coastal dunes

##### 4.1 Computational assumptions on the tree configuration

To evaluate the hydrodynamic resistance of trees against tsunamis, the tree configuration should be considered. We have already discussed the relationship of both trunk diameter  $d_0$  and tree height  $h_{tree}$  vs.  $N$  in Fig. 6 for idealized forests. In the following, we consider how the tree configurations of real forests might contribute fluid resistance against tsunami flows. We herein used measured data for coastal forests along several tsunami-prone coasts in Japan, acquired by the Japan Forestry Agency (2005).

The results of the  $d_0$  vs.  $N$  and  $h_{tree}$  vs.  $N$  relationships are shown in Fig. 8, in which the lines reflect the relationships indicated by Eq. (5) with  $C_1 = 0.06$  and  $C_2 = 5.6$ . It is evident that the empirical relationships

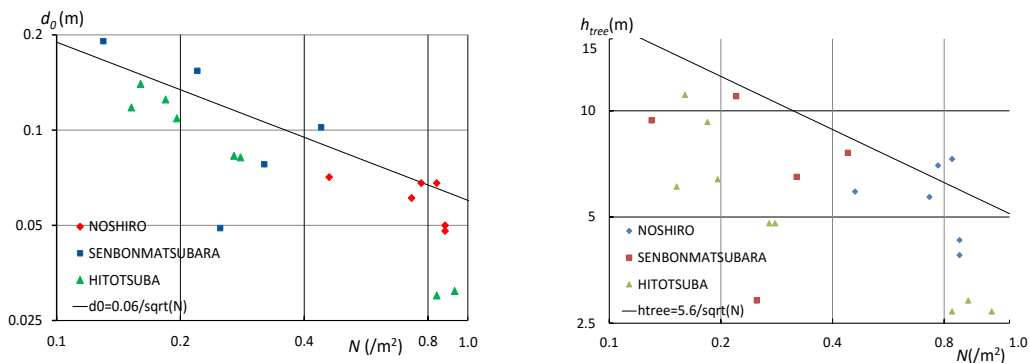


Fig. 8 Relationships between trunk diameter at breast height  $d_0$ , total tree height  $h_{tree}$ , and stand density  $N$

deduced from the idealized forests generally give greater values than measured. Furthermore, the measured data show a certain degree of scatter, especially for the  $h_{tree}$  vs.  $N$  relationship. These discrepancies are caused by differences between the real field and idealized situation, especially concerning the existence of human intervention in terms of forest management. Meanwhile, Fig. 9 shows clear correlation between branch height  $h_{br}$  (the height above ground of the lowest branch shoots) and total tree height  $h_{tree}$ . We use this relationship of  $h_{br} = 0.6h_{tree}$  for the following tree configuration modeling.

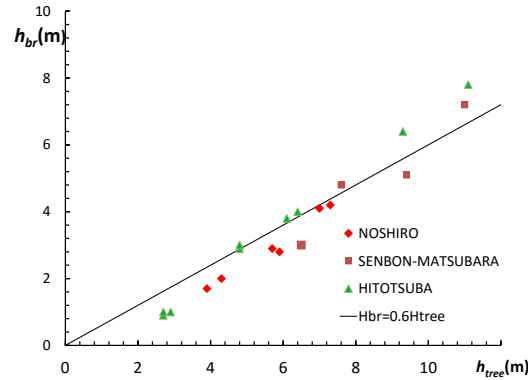


Fig. 9 Relationship between branch height  $h_{br}$  and total tree height  $h_{tree}$

In our model of tree configuration, the temporal change of stand density  $N$  is determined as a function of forest age  $\tau$  based on the Minowa's forest growth model. The temporal change of trunk diameter  $d_0$  is estimated using Eq. (5), with  $C_l = 0.06$  reflecting the change of  $N$ . The tree height  $h_{tree}$  can also be evaluated by the tree evolution model; however, because of the scatter shown in Fig. 8, we set  $h_{tree}$  to values of 3.0, 5.0, and 8.0 m for  $\tau=10, 20, 40$  years respectively with reference to a survey by Oda (1984) of a coastal black pine forest in Chiba Prefecture, Japan.

All other time-variant elements of trees can be modeled through the temporal change of stand density  $N$ . The upper part of a tree (crown area) comprises branches and leaves. Thus, the crown will contribute much greater fluid resistance when the tsunami height is greater than the branch height. Therefore, reasonable modeling of the projected area and occupied volume of the crown should be considered.

The projected areas of branches  $A_{br}$  and of leaves  $A_{lv}$  of a single tree are given based on the results of a survey by Noguchi et al. (2012) of black pine trees in coastal areas of Tohoku:

$$A_{br} = 32d_{br}^{1.7}, A_{lv} = 813d_{br}^{2.4} \quad (6)$$

where  $d_{br}$  is the trunk diameter at height  $h_{br}$ . Trunk diameter usually tends to decrease slightly with height; however, in this study,  $d_{br}$  is assumed the same as  $d_0$ . The product  $A_{br} + A_{lv}$  multiplied by  $N$  becomes the total projected area of the branch and leaves per unit ground area  $A_{total}^*$ . To express time-varying flow resistance attributable to the branches and leaves, in relation to an instantaneous tsunami surface level, the shape of the tree crown is assumed an inverse conical form (i.e., the upper circle with diameter  $w_{crown}$  is greater than the lower circle with diameter  $d_0$ ). The diameter  $w_{crown}$  is determined as the projected area that is equal to  $A_{br} + A_{lv}$ .

In the following calculations of tsunami flows in coastal forests, we set three forest cases: a young forest ( $\tau = 10$  yr), an intermediate-aged forest ( $\tau = 20$  yr), and an old forest ( $\tau = 40$  yr). Table 1 summarizes the computational conditions of the tree configurations for different forest ages.

Table 1 Computational conditions on tree configuration

$\tau$	$N$	$d_0$	$h_{tree}$	$A_{ink}$	$A_{br}$	$A_{lv}$	$A^*_{total}$
(year)	(m <sup>2</sup> )	(m)	(m)	(m <sup>2</sup> )	(m <sup>2</sup> )	(m <sup>2</sup> )	
10	0.626	0.076	3.0	0.144	0.400	1.680	1.300
20	0.422	0.092	5.0	0.276	0.554	2.650	1.350
40	0.228	0.126	8.0	0.605	0.946	5.640	1.500

#### 4.2 Basic equations for numerical calculation

One-dimensional mass and momentum conservation equations are used. The latter, is based on the non-linear shallow-water equation with the inclusion of terms for the drag and inertia associated with a group of trees:

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} = 0 \quad (7)$$

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x} \left( \frac{M^2}{D} \right) + gD \frac{\partial \eta}{\partial x} + \frac{gn^2}{D^{7/3}} M|M| + F_{tree} / \rho = 0 \quad (8)$$

where  $h = \eta + D$  is the total depth ( $D$ : still water depth,  $\eta$ : elevation from  $D$ ),  $M$  is line discharge,  $g$  is gravitational acceleration,  $\rho$  is water density, and  $n$  is Manning's bottom roughness (here, set to 0.25 s·m<sup>-1/3</sup>). The hydrodynamic force acting on the tree assembly per unit bottom area  $F_{tree}$  is given by the inertia and drag forces:

$$F_{tree} = \rho C_M a_1 \hat{h} d_0^2 N \frac{\partial M}{\partial t} + \frac{\rho}{2} C_D a_2 \hat{h} d_0 N \frac{M|M|}{D^2}, \quad (9)$$

where  $C_D$  is the drag coefficient,  $C_M$  is the inertia coefficient,  $V_{tree}$  is the occupied volume by trees (which is given by  $V_{tree} = a_1 \hat{h} d_0^2$ ;  $a_1$ : a coefficient), and  $A_{tree}$  is the projection area of trees (which is given by  $A_{tree} = a_2 \hat{h} d_0$ ;  $a_2$ : a coefficient). The latter two values are given as instantaneous values reflecting the relative height relationship between tsunami elevation  $\eta$ , tree branch height  $h_{br}$ , and total tree height  $h_{tree}$ , as illustrated in Fig. 10. Thus, if  $\eta < h_{br}$ , only the trunk contributes to the fluid force, if  $h_{br} < \eta < h_{tree}$ , the branches and leaves, which are assumed to take an inverse conical form, add to the fluid force, and if  $\eta > h_{tree}$ , the hydrodynamic force is assumed a constant value related to  $\eta = h_{tree}$ .

#### 4.3 Computational setup for beach topography

A model coast was set as shown in Fig. 11. The land region starts to extend from  $x = 8000$  m. A foreshore beach with a slope of 1/200 is set within the region  $x = 8000$  to 8200 m. A model coastal forest is assumed to grow from  $x = 8500$  m on a dune with a slope of 1/1000. The computations of tsunami flows were conducted under this basic topography (Case-1) for the three model forests, as shown in Table 1. Furthermore, topographic effects on tsunami flows were examined by modifying the beach shape from Case-1. As a first amendment, the foreshore slope was changed to 1/400 from 1/200 (Case-2). A second amendment placed a 1-m-high small triangular step on the dune at  $x = 8300$  m (Case-3). Finally, the location of the small step was shifted to  $x = 8200$  m (Case-4).

### 5. Computational results and discussion

Figure 12 illustrates the spatial and temporal variations of tsunami waves over the entire domain. In order to show the influence of the small step on the dune, the results of Case-2 (without step) and Case-3 (with step)

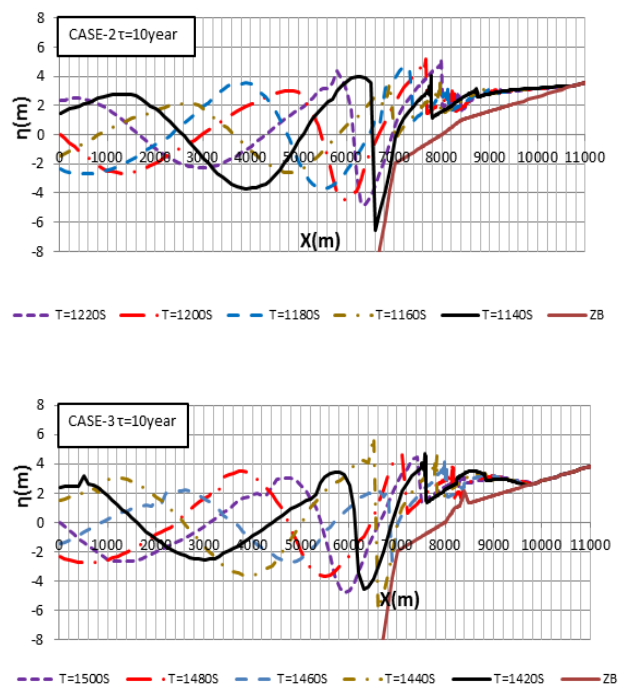


Fig.12 Tsunami propagation and run-up



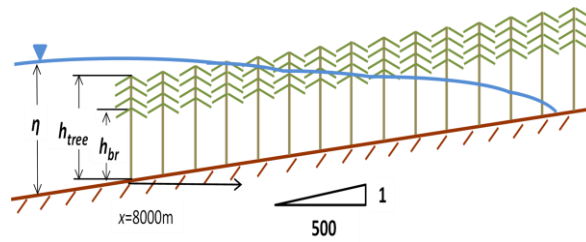


Fig. 10 Schematic description of flow resistance of trees relative to tsunami elevation

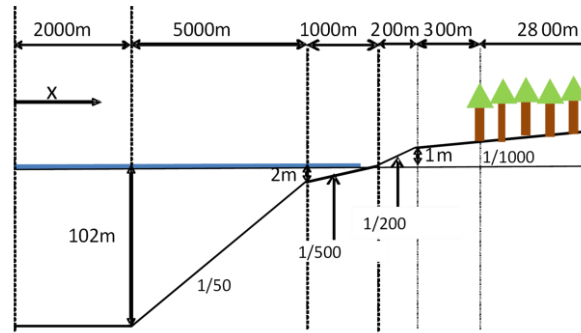


Fig. 11 Beach topography model (Case-1)

are compared. Non-negligible differences of tsunami wave fluctuation are found in the landward region from  $x = 8300$  m, where the step is placed. The maximum run-up length for Case-3 decreases because of hydraulic tree crown. As shown in this figure, the wave fluctuations in the offshore region of  $x = 0$  to  $8000$  m are minor; thus, in the following, the results of the landward region  $x = 8000$  to  $11,000$  m are discussed.

Figure 13 compares the results on the landward tsunami run-up waves for forests of different ages (Case-1). It is evident that the run-up fronts intrude furthest landward for the old forest case ( $\tau = 40$  yr). The intermediate-aged forest case ( $\tau = 20$  yr), shows little difference in intrusion distance in comparison with the old forest case. The stand density  $N$  for  $\tau = 40$  yr is lower than for  $\tau = 20$  yr because of self-thinning, although individual trees are larger because of their age. Such compensating effects are the reason for the similarity between the  $\tau = 40$  yr and  $\tau = 20$  yr cases. The intrusion of the run-up fronts for the young forest case ( $\tau = 10$  yr) is reduced significantly because of the large hydraulic resistance of the dense distribution of trees. In addition, more importantly, the crown (green shaded part in Fig. 11) of young trees is located at a lower height; thus, it contributes greater hydraulic resistance against tsunami flows. For the  $\tau = 20$  yr and  $\tau = 40$  yr cases, the branches and leaves are in higher positions, i.e., above the tsunami flow and thus, the total hydraulic resistance of these forests is reduced compared with the young forest case. resistance, mainly from the tree crown, because the step raises the tsunami water level to the height of the

Figure 14 illustrates the results for Case-2, where only the foreshore slope was changed from 1/200 to 1/400. The maximum intrusion distance increases by 400 m, in comparison with the upper panel in Fig. 11.

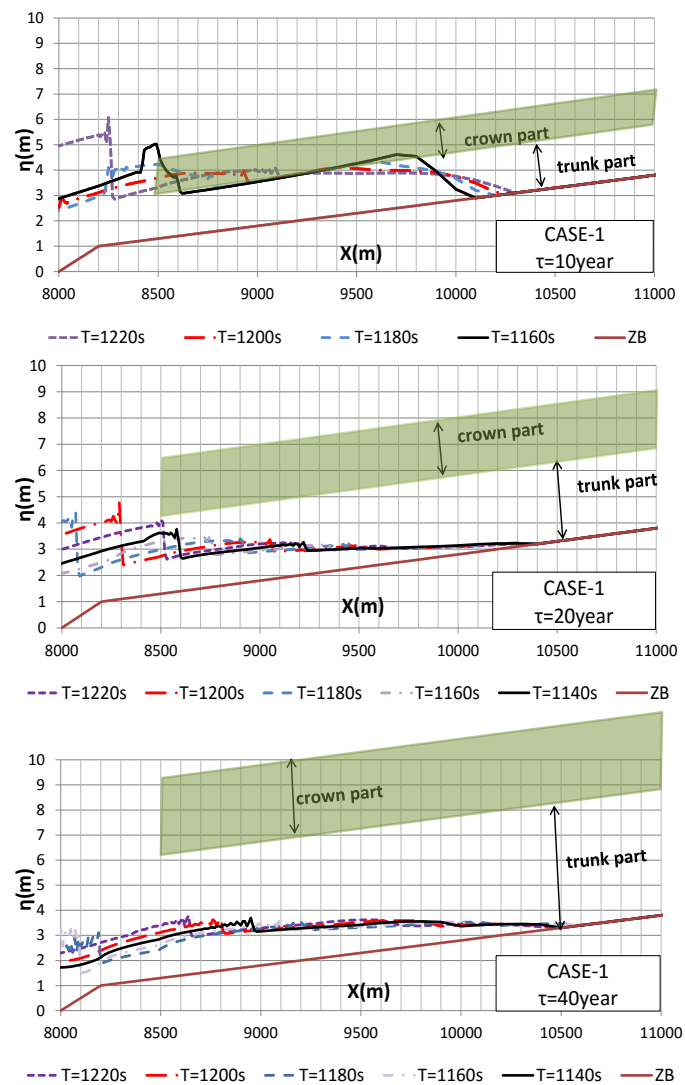


Fig.13 Water surface fluctuation of tsunami run-up waves (Comparison among different forest age cases; Case-1 topography)

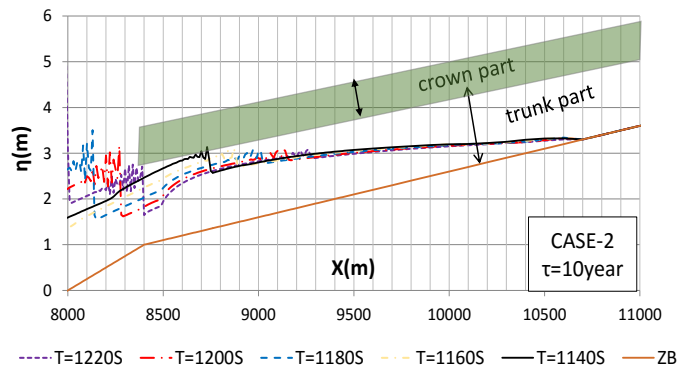


Fig.14 Water surface fluctuation of tsunami run-up waves (Changing foreshore slope to 1/400; Case-2 topography)

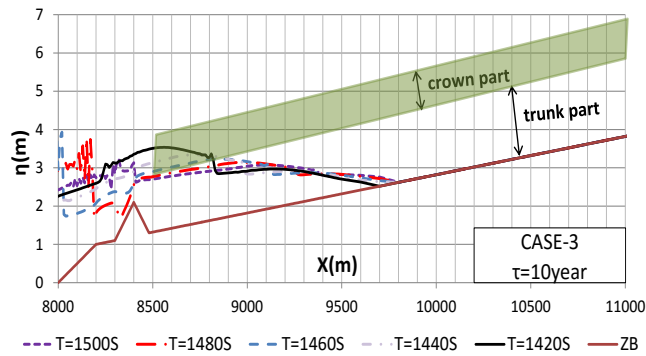


Fig. 15 Water surface fluctuation of tsunami run-up waves (Placing a triangular step; Case-3 topography)

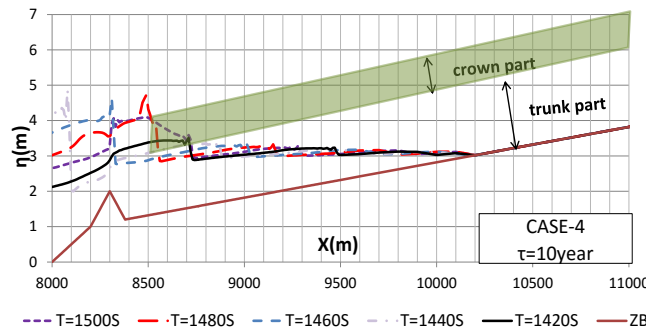


Fig. 16 ditto. (Changing location of a triangular step; Case-4 topography)

The highest tsunami elevation at  $x = 8500$  m is slightly greater than 3.0 m, i.e., it does not reach the tree crown. Therefore, the tsunami wave receives less resistance in Case-2, which results in the greater distance of intrusion.

Figures 15 and 16 show the results when a small sand dune with a triangular shape was placed at  $x = 8300$  m (Case-3) and  $8200$  m (Case-4), respectively. The situation of the tsunami wave reaching the crown area is different depending on the location of the step. It causes a difference in the maximum distance of intrusion of around 500 m between the two cases.

## 6. Conclusions

Through consideration of forest ecology and morphology, this study investigated the variation in performance of coastal forests of different ages in mitigating the effects of tsunamis. The model settings of coastal topography and forest characteristics were intended to approximate realistic conditions. A quantitative evaluation of the mitigating effects of coastal forests was undertaken by changing certain parameters.

The resistance against tsunami flows of an individual tree increases with forest age; however, stand density decreases through forest thinning as forest age increases. In addition, the crown of older trees is unlikely to contribute to flow resistance because tsunami flows do not extend that high. Because of these opposing effects, the maximum attenuation performance of a coastal forest will be realized at a certain year when the stand is of intermediate age.

Case studies were conducted by changing certain parameters of beach topography, such as the foreshore slope or by placing a small step on the dune. These changes cause tsunami flows to reach the tree crown area, which possesses greater resistance, resulting in noticeable differences in the maximum tsunami run-up.

The following subjects remain possibilities for future studies.

The present model assumed the trees are rigid and strong, and not swayed or broken by the tsunami force. In reality, however, trees are destroyed when the tsunami force becomes large. Therefore, the present model should be extended to include the material strength of trees.

The present model of forest growth assumed an idealized forest composed of a number of trees of a single species of the same age under uniform environmental conditions. The model also assumed the forest is natural without human management. However, most coastal forests are under human management and therefore, the model should be extended to account for more realistic forest conditions.

For the practical use of coastal forests as a tsunami-protection facility, the forest must be designed to attenuate the predicted effects of tsunamis for a certain period considering the time-varying functionality associated with forest growth. To maintain desired performance levels, optimal forest management to control stand density by artificial thinning or planting, and to regulate tree morphology by branch cutting should be considered.

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