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# WAVE HEIGHT DISTRIBUTIONS IN THE SURF ZONE: IMPLICATIONS FOR SURF ZONE MODELLING

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**Abstract**

Wave height distributions in the surf zone on natural beaches have been shown to be significantly different to a Rayleigh distribution with waves more narrowly distributed and better described by a Weibull distribution with an exponent that varies with depth. Modified wave height distributions are incorporated into a standard parametric wave height transformation model with the model outputs showing changes in the location and intensity of wave energy dissipation with wave height distributions narrower than Rayleigh distributions showing later and more condensed energy dissipation and models with wave height distributions wider than Rayleigh distributions showing the reverse. Model outputs are compared to a large field dataset with mixed results with reduced errors for some locations but increased errors for others.

**Key words:** surf zone, wave heights, wave height distributions, Rayleigh distribution, Weibull distribution, parametric modelling

## 1. Introduction and background

Accurate modelling of processes in the surf zone is a crucial factor for effective coastal management. Quality surf zone models can be used to drive runup, setup, and sediment transport models to enable prediction of coastal change in response to changing offshore conditions. Parametric wave transformation models are widely used in cross-shore morphological models due to their computational efficiency (Baldock *et al*., 1998). In order to use parametric models to predict random wave behaviour, however, assumptions must be made about the wave height distribution function and the fraction of broken waves (Battjes and Janssen, 1978; Thornton and Guza, 1983; Baldock *et al*., 1998; Ruessink *et al*., 2003).

The most common parametric descriptor of ocean waves, including waves within the surf zone, is the Rayleigh distribution. The Rayleigh probability density function is given by:

$$f(x) = 2\frac{H}{H_{rms}} exp\left[-\left(\frac{H}{H_{rms}}\right)^2\right] \tag{1}$$

and the cumulative distribution function is given by:

$$F(X) = P(x < X) = 1 - exp\left[-\left(\frac{x}{H_{rms}}\right)^2\right] \tag{2}$$

where $H_{rms}$ is the root mean square wave height (see Nielsen, 2009, for further details, pp. 56-61; thick solid line in Figure 1). Since the observations of Longuet-Higgins (1952), which showed that for a linear model of waves with a narrow energy spectrum the heights of the waves fit a Rayleigh distribution well, numerous field observations have also shown that zero-crossing wave heights are approximately Rayleigh distributed in the surf zone (*e.g.*, Thornton and Guza, 1983; Massel, 1996). If waves are Rayleigh distributed, all the characteristic wave heights ($H_{rms}$, $H_{sig}$, $H_{1/10}$, etc.) can be calculated from the standard deviation of the water surface elevation using known constants. Given the evidence that the wave height

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distribution can be well described by the Rayleigh distribution both offshore and in the surf zone, the Rayleigh distribution is widely used in numerous random wave transformation models, and therefore needs to be a correct descriptor of the actual wave height distribution if wave transformation models are to provide accurate results. Several recent studies, however, have challenged the use of the Rayleigh distribution as an accurate descriptor of waves in the surf zone.

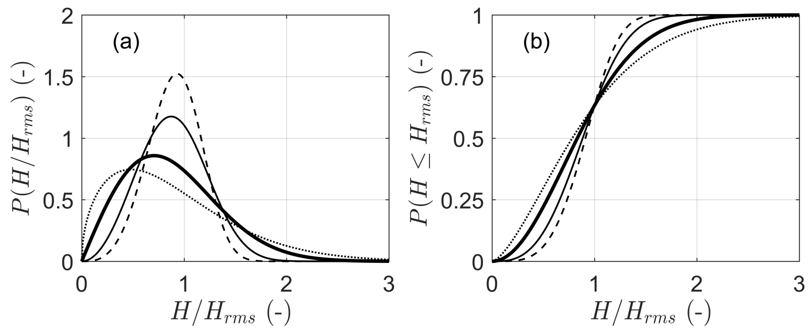


Figure 1. Examples of the (a) probability density function and (b) cumulative distribution function for the Rayleigh distribution (thick solid line), and Weibull distribution with *p*=1.5 (dotted line), *p*=3 (thin solid line), and *p*=4 (dashed line). The Rayleigh distribution is equivalent to a Weibull distribution with *p*=2.

The pioneering fieldwork of Thornton and Guza (1983) measured wave heights and obtained probability distributions at various locations through the surf zone for both unbroken and broken waves. Visual observed showed the Rayleigh distribution to give good overall estimates of wave height statistics, however, it slightly over predicted the number of waves in the tail of the distribution. Ting (2001) examined the distribution of laboratory wave heights for broad-banded irregular waves and also observed a departure from the Rayleigh distribution in shallower water depths. When less than 20% of waves were breaking, waves were observed to be better predicted by a Beta-Rayleigh distribution than a Rayleigh distribution but for the remainder of the surf zone, neither distribution was found to be adequate. Battjes and Groenendijk (2000) developed a predictive model for local wave height distributions in a laboratory surf zone. Their model was composed of two Weibull distributions and used local wave energy, depth, and bottom slope to derive the distribution. The Weibull probability density function is given by:

$$f(x) = p\left(\frac{H}{H_{rms}}\right)^{p-1} exp\left[\ \left(\frac{H}{H_{rms}}\right)^{p}\right] \tag{3}$$

and the cumulative distribution function is given by:

$$F(X) = P(x < X) = 1 - exp\left[-\left(\frac{x}{H_{rms}}\right)^{p}\right] \tag{4}$$

A Rayleigh distribution corresponds to a Weibull distribution where the exponent *p*=2 (see Figure 1). van Vledder *et al*. (2013) compared three distributions with laboratory data. They found that neither the Rayleigh, nor a distribution developed by Glukhovskiy (1966) and Klopman (1996), nor the distribution developed by Battjes and Groenendijk (2000) were valid in shallow water depths.

More recently, Power *et al*. (2016) used a large field data set to show that the wave height distribution in the surf zone of natural beaches varied significantly from a Rayleigh distribution. They showed that in over 50% of data runs, wave height distributions were significantly different to a Rayleigh distribution with the majority of distributions being more narrowly distributed than a Rayleigh distribution (e.g., Figure 2). Distributions were observed to become narrower with decreasing depth with fewer extreme values. To characterise the observed distributions, data were fitted to a Weibull distribution optimising the exponent, *p* (Figure 1; Equations 3 and 4). An average value of *p*=2.4 was shown for the full dataset, further supporting the observation that wave heights in the surf zone are more narrowly distributed than a Rayleigh distribution, however, values of *p* ranged from 1.5 to 4 (Power *et al*., 2016). They showed that the optimal value of the power exponent (i.e., the width of the Weibull distribution) was correlated with local mean depth and with depth normalized by offshore wave height and also showed that distributions of wave heights become narrower (i.e., the exponent increases) as depth decreases such that:

$$p = -0.45h/H_o + 2.81 \tag{5}$$

where $h$ is depth and $H_o$ is offshore wave height, and

$$p = (3.83 - 2.78h)/(h^2 + 2.72) + 2 \tag{6}.$$

Due to the requirement that offshore wave heights be Rayleigh distributed (i.e., $p$=2), Equation 5 is only applied for $h/H_o \leq 1.8$ to ensure that $p$ is restricted to $p \geq 2$.

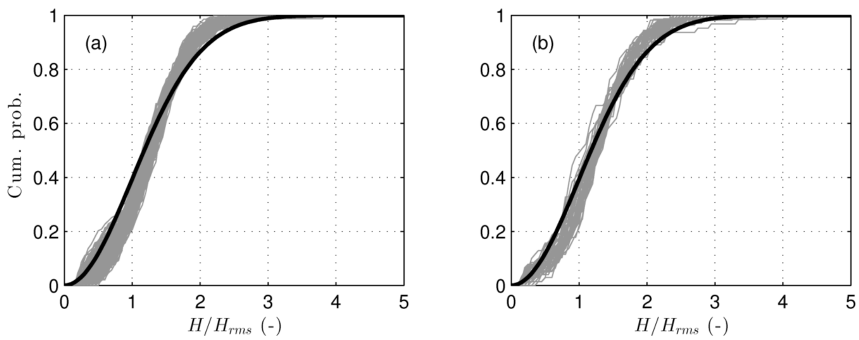


Figure 2. Cumulative probability distribution of normalised wave height, $H/H_{rms}$, for each data run (grey lines) compared to the cumulative Rayleigh distribution function (black line) for two example deployments: (a) Moreton Island 8 December 2008, and (b) The Spit, Gold Coast, 12 March 2009 (from Power *et al*., 2016).

As noted in several papers, significant differences exist between random wave model predictions and field data (see Apotsos *et al*., 2008, for a review). A proportion of this error may be due to the assumption in some random wave models that the probability distribution of wave heights in the surf zone conforms to a Rayleigh distribution. Given the recent research that suggests that the distribution of wave heights departs from a Rayleigh distribution in the surf zone, particularly in the shallower depths of the surf zone, it is of interest to investigate the impact of the wave height distribution on the results of widely used parametric wave transformation model.

## 2. Parametric wave transformation modelling

### 2.1. *Modified wave height distributions*

To assess the impact of wave height distribution on cross-shore wave height dissipation models, the Alsina and Baldock (2007) parametric wave transformation model was run with varying wave height distributions in place of the standard Rayleigh distribution. This model was chosen as it is based on the Battjes and Janssen (1978) model which is a widely used parametric wave model and is the basis of several engineering models. Changing the wave height distribution in the model alters the predicted wave energy dissipation due to a change in the fraction of broken waves. By incorporating a distribution other than the Rayleigh distribution into the dissipation formulation used in this random wave transformation model, cross-shore wave height transformation of non-Rayleigh distributed waves can be predicted.

Initially, the wave height transformation model was run with a Weibull distribution (Equations 3 and 4) with fixed values for $p$ ($p$=1.5, 2, 3, and 4) on a plane beach with a 1/30 slope and initial model conditions of $H_{rms}$ = 2 m, $T$ = 8 s, and $h(x = 0 \text{ m})$ = 10 m. Results showed that changing the value of $p$ altered the patterns of wave energy dissipation with higher values of $p$ showing dissipation occurring later in the model, i.e., energy dissipation starting in shallower water depths, being concentrated over a small depth range, and occurring at greater rates (Figure 3). This results in larger wave heights and therefore larger wave height to water depth ratios ($\gamma$ values) in the surf zone. Conversely, lower values of $p$ showed showed a more widespread region of wave breaking with energy dissipation starting in deeper water depths and being more evenly spread across a larger range of depths which results in lower wave heights and $\gamma$ values across the profile (Figure 3).

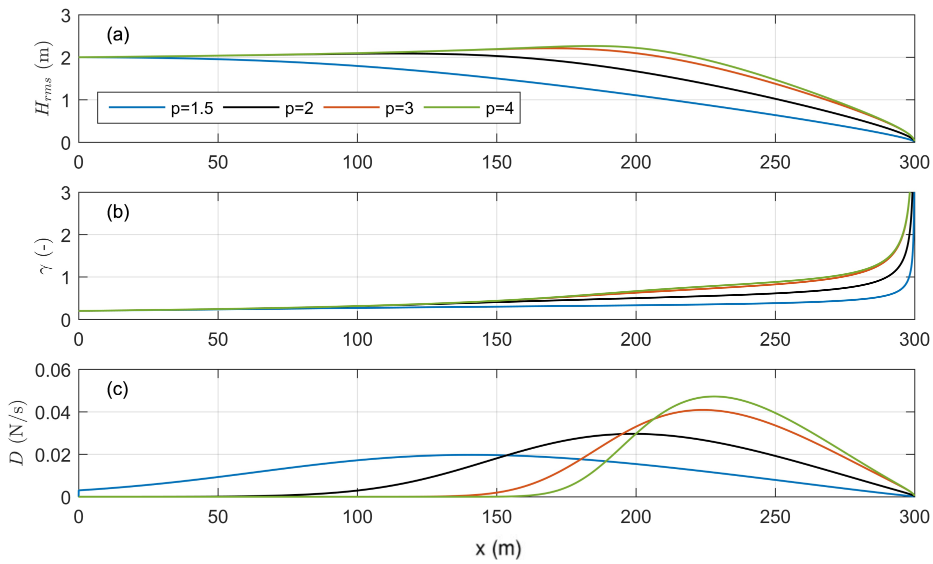


Figure 3. The effect of varying the value of $p$ on (a) cross shore wave height transformation ($H_{rms}$), (b) gamma values ($\gamma$), and (c) energy dissipation ($D$) for a plane beach profile with $\tan\beta = 1/30$ for four different values of $p$: $p = 1.5$ (blue line), $p = 2$ (black line), $p = 3$ (red line), and $p = 4$ (green line). The model was run with initial conditions of $H_{rms} = 2$ m, $T = 8$ s, and $h(x = 0 \text{ m}) = 10$ m.

The variation shown in Figure 3 occurs due to the variation in the fraction of broken waves, $Q_b$, that results from varying the wave height distribution. For the four values of $p$ shown in Figure 3, the corresponding fraction of broken waves is shown in Figure 4. As $p$ increases, the fraction of broken waves is more sensitive to small changes in $H_b$ or $H_{rms}$ due to the narrow nature of the wave height distribution.

### 2.2. *Cross-shore varying wave height distributions*

Given that the results of Power *et al*. (2016) show that wave height distributions narrow with decreasing depth and that the optimal value of the power exponent of the Weibull distribution (i.e., the width of the distribution) correlates with both local mean depth and with depth normalized by offshore wave height, it is of interest to investigate how varying the wave height distribution with varying depth affects wave height transformation.

Using the two most accurate best fit lines obtained by Power *et al*. (2016) as shown in Equations 5 and 6 and using the method described in Section 2.1, the Alsina and Baldock (2007) model was run with depth-varying wave height distributions in place of the standard Rayleigh distribution on a plane beach with a 1/30 slope and initial model conditions of $H_{rms} = 2$ m, $T = 8$ s, and $h(x = 0 \text{ m}) = 10$ m (Figure 4). For the model with depth varying wave height distributions that are dependent on depth normalized by offshore wave height (blue lines in Figure 4), modelled wave heights in shallow depths are larger than those predicted using a Rayleigh distribution with energy dissipation occurring later. Consequently, $\gamma$ values are also greater in the shallow depths of the surf zone than for waves that are Rayleigh distributed. For the model with depth varying wave height distributions that are dependent on depth only, wave height transformation is significantly different to that predicted with a Rayleigh distribution due to the equation predicting wave height distributions wider than a Rayleigh distribution in the outer surf zone and shoaling zone as per the observations of Power *et al*. (2016). However, increases in $p$ closer to the shore still result in increased wave heights and $\gamma$ values in the shallow water depths of the surf zone which therefore also results in relatively increased rates of energy dissipation when compared with Rayleigh distributed waves.

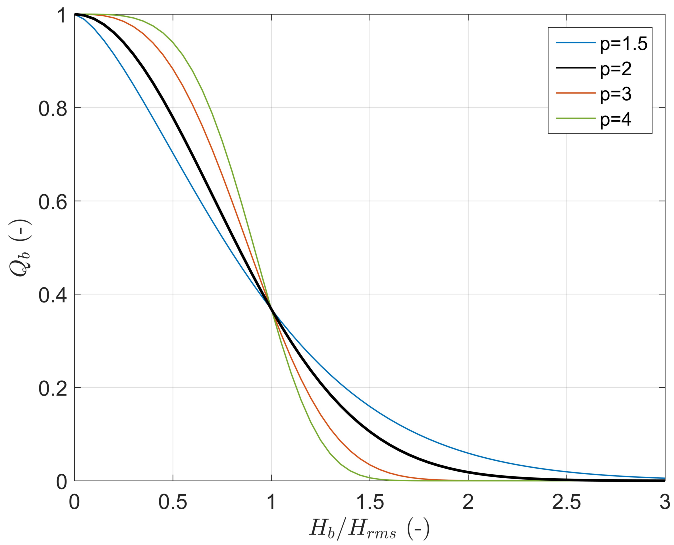


Figure 3. The effect of varying the value of $p$ on the fraction of broken waves ($Q_b$) for four different values of $p$: $p$ = 1.5 (blue line), $p$ = 2 (black line), $p$ = 3 (red line), and $p$ = 4 (green line).

## 3. Comparison with field data

To assess the ability of the Alsina and Baldock model to predict field observations using modified wave height distributions, the model was compared to a subset of the dataset used in Power *et al*. (2016). Only a data runs with four or more pressure transducers in the surf zone were used in this subset to ensure at least three data points for model-data comparison (as the first point was used to initiate the model). A total of 203 data runs were suitable for model-data comparisons which consisted of a total of 1613 individual pressure records with an average of just under 8 pressure sensors in each data run. For further details on the dataset, see Power *et al*. (2010, 2016). The two models described in Section 2.2 were modelled using data from the offshore most pressure sensor to initiate the model and modelled values for $H_{rms}$ and $\gamma$ were compared to measured values. Measured values were also compared to modelled values obtained using a Rayleigh distribution to describe the wave heights.

Example model outputs for the three wave height transformations are shown in Figure 5 along with measured wave heights and $\gamma$ values. Results were mixed with the modified distributions performing better (as indicated by lower root mean square error values) than the standard Rayleigh distribution for some data runs but worse for others. As with the models run on plane slopes (Section 2.2), the modelled data using distributions that vary with depth and depth normalized by offshore wave height have larger wave heights and $\gamma$ values in the shallow water depths of the surf zone with greater rates of energy dissipation at the landward edge of the surf zone when compared to a model with Rayleigh distributed waves. Root mean square error (RMSE) values for the six model runs shown in Figure 5 are shown in Table 1. Variations between RMSE values for individual data runs are typically small but range up to a factor 3 for some data runs.

Model-data comparisons for the dataset described above showed mixed results. Table 2 shows daily averaged root mean square error (RMSE) values for $H_{rms}$ and $\gamma$ for all model runs analysed for three different wave height distributions. For some locations, change the wave height distribution used in the model to a depth varying distribution resulted in significantly better outcomes with improvements in RMSE values for both $H_{rms}$ and $\gamma$ values of up to 30%. However, for other locations, model results with depth varying wave height distributions showed higher RMSE for both $H_{rms}$ and $\gamma$ values. Change in model performance, as described by a percentage change in RMSE for both $H_{rms}$ and $\gamma$ values were standard beach parameters, such as Iribarren number and beach slope, but no clear correlations between change in model performance and these parameters were observed (not shown).

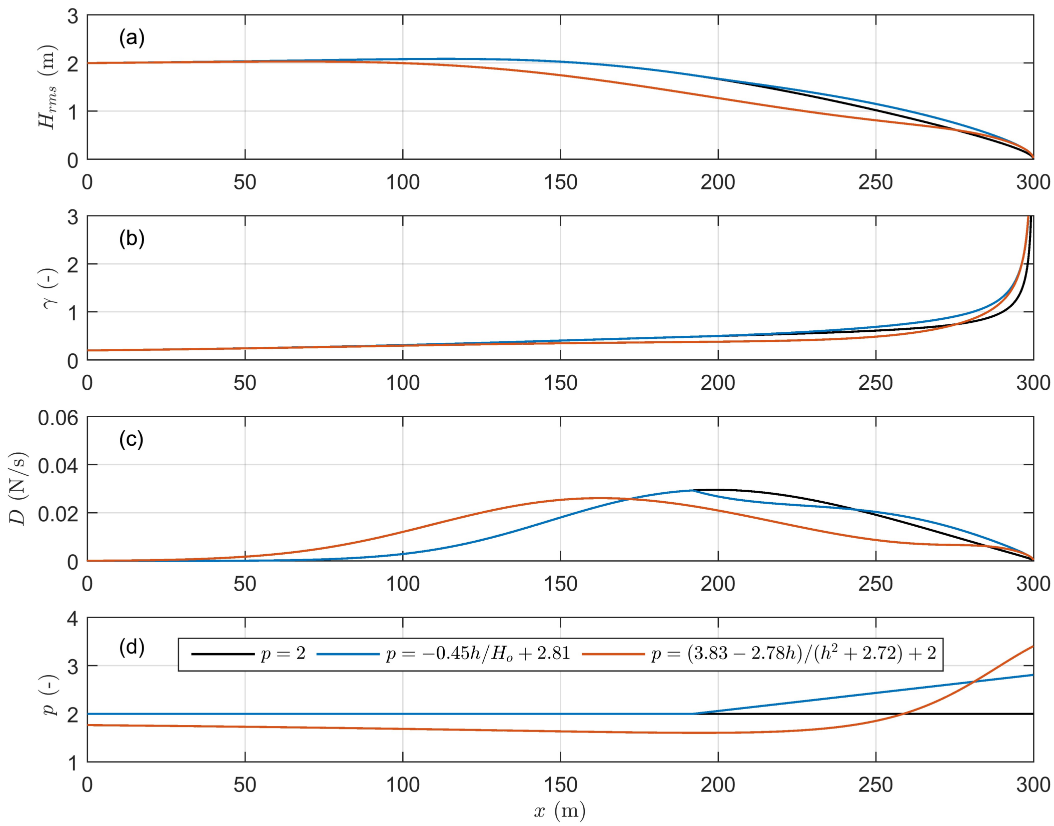


Figure 4. The effect of varying the value of $p$ on (a) cross shore wave height transformation ($H_{rms}$), (b) gamma values ($\gamma$), and (c) energy dissipation ($D$) for a plane beach profile with $\tan\beta = 1/30$ for one fixed and two varying values of $p$ as shown in (d): $p = 2$ (i.e., a Rayleigh distribution; black line), $p = -0.45h/H_o+2.81$ for $h/H_o \leq 1.8$ (blue line), and $p = (3.83\text{-}2.78h)/(h^2+2.72)+2$ (red line). The model was run with initial conditions of $H_{rms} = 2$ m, $T = 8$ s, and $h(x = 0 \text{ m}) = 10$ m.

Table 1. Root mean square error (RMSE) values for $H_{rms}$ and $\gamma$ for the model runs shown in Figure 5 for three different wave height distributions: a Rayleigh distribution (i.e., $p$=2), a distribution that varies with depth normalised by offshore wave height ($p=f(h/H_o)$), and a distribution that varies with depth ($p=f(h)$). For each data run the lowest RMSE for both $H_{rms}$ and $\gamma$ are shown in italics. See text and Figure 5 for further details.

| Date | Run | RMSE ($H_{rms}$; $p$=2) | RMSE ($\gamma$; $p$=2) | RMSE ($H_{rms}$; $p=f(h/H_o)$) | RMSE ($\gamma$; $p=f(h/H_o)$) | RMSE ($H_{rms}$; $p=f(h)$) | RMSE ($\gamma$; $p=p=f(h)$) |
|---|---|---|---|---|---|---|---|
| 11/05/2004 | 21 | 0.071 | 0.52 | 0.030 | 0.43 | *0.025* | *0.42* |
| 24/04/2007 | 1 | 0.14 | 0.40 | *0.092* | *0.31* | 0.099 | 0.33 |
| 10/12/2007 | 32 | 0.039 | 0.064 | *0.026* | *0.040* | 0.028 | 0.042 |
| 11/12/2007 | 29 | *0.018* | *0.024* | 0.030 | 0.039 | 0.026 | 0.032 |
| 11/12/2007 | 31 | 0.043 | 0.051 | *0.034* | *0.038* | 0.035 | 0.040 |
| 08/12/2008 | 19 | *0.053* | *0.11* | 0.083 | 0.20 | 0.081 | 0.19 |

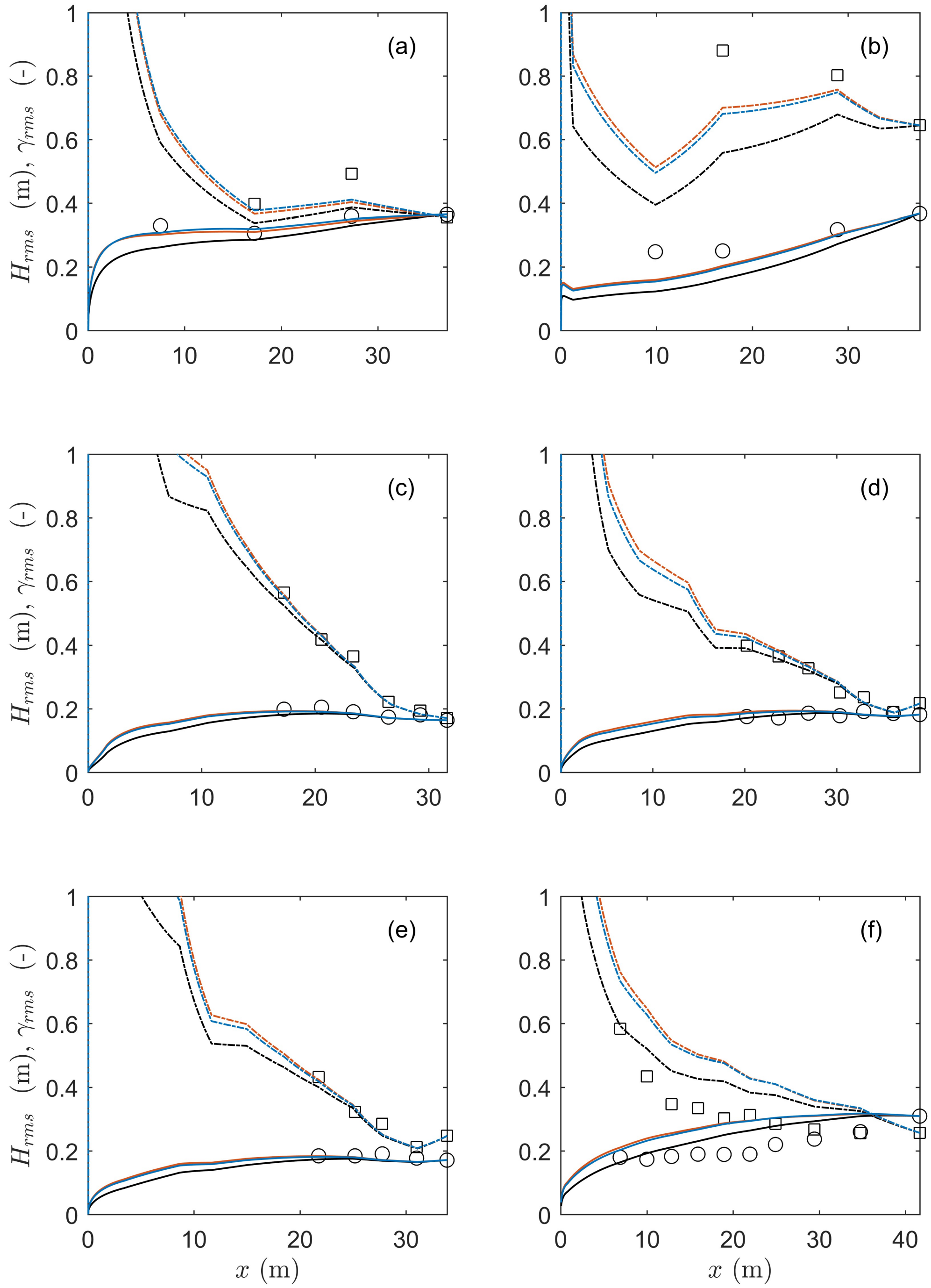


Figure 5. Comparison of measured data on wave heights (open circles) and $\gamma$ values open squares) with the cross-shore wave height transformation model predictions of wave heights (solid lines) and $\gamma$ values (dashed lines) for six data runs (see Table 1 for details). Each panel shows model results from models with three different wave height distributions: a Rayleigh distribution, i.e., a fixed value of $p = 2$ (black lines), a distribution that varies with depth normalised by offshore wave height $p = -0.45h/H_o+2.81$ for $h/H_o \leq 1.8$ (blue lines), and a distribution that varies with depth $p = (3.83-2.78h)/(h^2+2.72)+2$ (red lines).

Table 2. Daily averaged root mean square error (RMSE) values for $H_{rms}$ and $\gamma$ for all model runs analysed for three different wave height distributions: a Rayleigh distribution (i.e., $p$=2), a distribution that varies with depth normalised by offshore wave height ($p=f(h/H_o)$), and a distribution that varies with depth ($p=f(h)$). See text for further details. Note that the number of data runs differs between days.

| Date | Iribarren number (-) | RMSE ($H_{rms}$; $p$=2) | RMSE ($\gamma$; $p$=2) | RMSE ($H_{rms}$; $p= f(h/H_o)$) | RMSE ($\gamma$; $p= f(h/H_o)$) | RMSE ($H_{rms}$; $p=f(h)$) | RMSE ($\gamma$; $p= p=f(h)$) |
|---|---|---|---|---|---|---|---|
| 06/05/2004 | 0.22 | 0.075 | 0.15 | 0.053 | *0.12* | *0.051* | 0.13 |
| 07/05/2004 | 0.11 | 0.067 | 0.18 | *0.040* | *0.13* | 0.044 | 0.14 |
| 11/05/2004 | 0.19 | 0.042 | 0.24 | 0.042 | *0.17* | *0.032* | 0.18 |
| 16/11/2004 | 0.43 | *0.027* | *0.12* | 0.044 | 0.084 | 0.035 | 0.091 |
| 17/11/2004 | 0.61 | *0.043* | 0.34 | 0.050 | *0.32* | 0.044 | 0.34 |
| 24/04/2007 | 0.09 | 0.18 | 0.45 | 0.13 | 0.38 | *0.12* | *0.37* |
| 10/12/2007 | 0.23 | *0.043* | *0.084* | 0.048 | 0.095 | 0.045 | 0.090 |
| 11/12/2007 | 0.22 | *0.041* | *0.048* | 0.043 | 0.050 | 0.043 | 0.051 |
| 01/10/2008 | 0.20 | 0.039 | *0.36* | *0.038* | 0.37 | *0.038* | 0.37 |
| 18/11/2008 | 0.38 | *0.043* | *0.20* | 0.051 | 0.26 | 0.051 | 0.27 |
| 07/12/2008 | 0.15 | *0.064* | *0.095* | 0.083 | 0.13 | 0.090 | 0.14 |
| 08/12/2008 | 0.19 | *0.058* | *0.13* | 0.087 | 0.21 | 0.087 | 0.21 |
| 09/12/2008 | 0.21 | *0.032* | *0.067* | 0.061 | 0.14 | 0.058 | 0.13 |
| 10/03/2009 | 0.12 | 0.033 | *0.12* | 0.032 | 0.20 | *0.028* | 0.19 |
| 11/03/2009 | 0.21 | 0.037 | 0.055 | *0.026* | *0.039* | 0.031 | 0.046 |
| 12/03/2009 | 0.22 | *0.029* | 0.14 | 0.044 | 0.11 | 0.033 | *0.13* |
| Total dataset | | 0.046 | 0.14 | 0.054 | 0.15 | 0.052 | 0.15 |

## 4. Conclusions

Modifying the wave height distribution function that is incorporated into a standard random wave transformation model significantly alters the model results with the location and intensity of energy dissipation changing with changes in the wave height distribution. Model runs with wave height distributions wider than a Rayleigh distribution result in wave energy dissipation occurring over a wider area and lower $H_{rms}$ and $\gamma$ values when compared to Rayleigh distributed waves on the same profile. In contrast, model runs with wave height distributions narrower than a Rayleigh distribution have concentrated regions of wave energy dissipation which results in large wave heights and $\gamma$ values, particularly in the shallow depths of the surf zone. Comparisons of parametric wave height models with depth varying wave height distributions to field data show reduced root mean square errors for some datasets but increased errors for others. No clear correlations between improvement in model performance and standard beach parameters such as Iribarren number and beach slope were observed.

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