UNCERTAINTY ASSESSMENT IN COASTAL MORPHOLOGY PREDICTION WITH A BAYESIAN NETWORK

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Abstract

In the present time of sea-level rise and climate change a global shift has occurred toward sandy coastal protection measures and Building with Nature. These type of protection measures impose extra uncertainty on the instantaneous state of the coastal system over time for which present deterministic forecasting techniques are not capable of providing necessary information on uncertainties and hence could display a false sense of accuracy and skill. At present in long term morphological modeling a full systemic approach for uncertainty assessment has not yet been applied. This paper investigates the use of a Bayesian Network as a tool for uncertainty assessment in decadal scale morphological modeling for the evolution of a mega nourishment at the Dutch North-Holland coast, the Hondsbossche Dunes (HBD). The Bayesian Network is trained with an existing set of model data and field data of one year bed development. The Bayesian Network successfully transfers the bandwidth in input variables, model uncertainty and calibration uncertainty to an uncertainty bandwidth around the output parameter of choice.

Key words: Bayesian Network, uncertainty, morphodynamics, numerical modelling, Building with Nature

1. Introduction

A good prediction of coastal behavior is important in the present time of sea-level rise and climate change. Sandy coastal protection measures have been increasingly proposed in the last decades for locations with receding coastlines. The recent adaptation of the Building with Nature paradigm for sandy interventions (De Vriend et al, 2014) imposes extra uncertainty on the instantaneous state of the coastal system over time as it proposes using natural dynamics to our benefit. With little knowledge on the magnitude of the uncertainties in our predictions of coastal evolution, the long-term strategy development and design of projects is impeded.

At present, deterministic forecasting on decadal time-scales with 2DH-models is common practice in coastal modeling for wave and tide exposed regions (i.e. Luijendijk et al 2017). These (traditional) prediction techniques rely on individual deterministic predictions that are not capable of providing necessary information on uncertainties and hence could display a false sense of accuracy and skill. Probabilistic forecasts are less common and are thought to be computationally expensive, yet could be effective for decision making under uncertainty.

In the last decade the first probability assessments have emerged in certain fields of coastal modeling. Baart et al. (2011) and Baart (2013) have shown that confidence bounds on morphological evolution could be obtained by incorporating stochastics of dune strength (Figure 1, left). Plant et al. (2011a;2011b) have successfully applied a stochastic approach in modeling the hydrodynamics of surfzone processes. These findings suggest that stochastic modeling of the full profile in the near future would be possible (Figure 1, right).

One of the techniques to incorporate uncertainty into predictions is the Bayesian Network (BN hereafter). Plant et al. use such a BN, revealing it can accurately transfer input uncertainty (i.e. offshore

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wave parameters) to output uncertainty (inshore hydrodynamics). Similarly, Gutierrez et al. (2011) use a BN to make probabilistic predictions of shoreline retreat in response to different future sea level rise rates. They show that BNs can make quantitative, probabilistic predictions that can be applied to coastal management decisions. In meteorology stochastic approaches are far more common. Coelho et al. (2006) show that a BN can be used to improve uncertainty estimates of rainfall predictions.

The objective of the current study is to do an initial investigation on the use of a BN for morphological modeling of surfzone evolution. It is specifically a step towards a full systemic approach for uncertainty assessment in long term morphological predictions (Figure 1, right), examining BN as one of many possible concepts for probabilistic assessments.
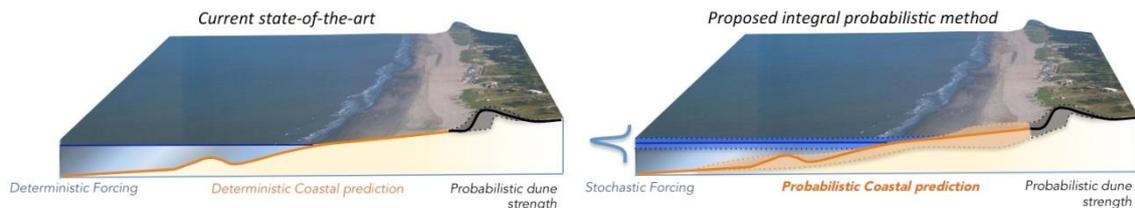

Figure 1: Schematic on stochastic model simulations. Traditional, determinist forcing of morphological models resulting in deterministic predictions of coastal evolution and including stochastics only for dune strength (left panel) versus the probabilistic assessment of morphological changes of the full profile (right panel), including stochastic wave forcing and initial bed conditions. Shading indicates a confidence interval around forecasted values.

2. Methodology

2.1 Approach

To investigate the use of BNs as a tool for uncertainty assessment in decadal scale morphological modeling a simple BN is introduced that propagates 1) intrinsic uncertainties (i.e. wave forcing), 2) uncertainties related to model limitations (i.e. cross shore profile development) and 3) model uncertainties through the process chain. The BN is constructed and used to quantify model prediction uncertainty for a case study beach, the Hondsbossche Dunes mega nourishment at the Dutch North-Holland coast. The network is trained with an existing set of model – and field data and the BN applicability for uncertainty propagation is investigated with one year of available observations.

This training data set consists of a set of sensitivity calculations testing individual relationships between uncertain parameters (e.g. wave forcing magnitude) and response (volume loss in the area). These calculations were conducted before construction of the HBD (Kroon et al, 2015) as part of the prognosis for required coastal maintenance in the 20 years after construction. In these calculations several sources of uncertainty in a 2DH process-based model approach were identified, quantified and used in a Monte Carlo analysis.

In this study we apply the previously computed sensitivity calculations in a network, such that it can be used to forecast the bulk volume loss at the nourished site after training. For the implementation of the Bayesian Network the software package Netica (Norsys, 2016) is used.

To test the validity of the BN, it is tested for consistency with previously known dependencies (i.e. a face validity check) and tested against observations from the first year after construction (i.e. evidence testing).

2.2 Hondsbossche dunes case study beach

The case study beach used for the evaluation of the Bayesian Network is the recent reinforcement of the Hondsbossche and Pettemer Sea Defence, called the Hondsbossche Dunes (HBD). The project consists of a mega nourishment of 35 million m^3 situated along the Dutch North-Holland coast between IJmuiden and Den Helder (Figure 2). The alongshore length is approximately 12 km and the nourishment extends 500 m seaward with respect to the adjacent coastline, Figure 2. The nourishment is situated in a wave dominated, micro-tidal environment on a coastline that has a history of structural erosion.

Figure 2: Hondsbossche and Pettemer Zeewering, a sandy coastal flood protection near Petten, the Netherlands. The original, hard protection in 2005 (left) and the new sandy coastal protection in 2015 (right).

After installation of the nourishment in 2015, the sub-aqueous and sub-aerial bed development of the HBD is monitored along 11 transects with a monthly interval.

2.3 Bayesian Networks

A BN is a graphical representation of a high dimensional probability distribution of a finite set of discrete variables (Jensen and Nielsen, 2007). At the core of the Bayesian network is Bayes theorem, which can be used to propagate evidence through the network in a predictive (forward) or diagnostic (backward) manner. The network needs to be trained on large sets of (model/observational) data. After training all information of the model calculations is contained in the model.

Bayes theorem is given as:

$$p(F|O) = \frac{p(O|F)p(F)}{p(O)} \tag{1}$$

in which the left-hand side represents the updated conditional probability of a forecast, *F*, given a certain set of observations, *O*.

Bayesian Networks are used for different purposes such as the inference of the belief in a certain event given certain evidence, determination of the most likely explanation of a certain event and for uncertainty propagation (Den Heijer, 2013; Plant et al 2011a; Plant et al 2011b). A Bayesian Network differs from a Neural Network in the way that in a Bayesian Network not all nodes are related. Therefore it is possible to include knowledge on the relations and the direction of these relations between parameters in the network.

2.4 Network formulation

A basic BN is constructed using four input parameters with uncertainty exemplary for uncertainty in forcing, morphology, model uncertainty and model calibration. The coastal state indicator under consideration is the bulk volume loss over the project bounds in alongshore direction (ΔV). The four contributions to uncertainty under consideration, which are quantified in Kroon et al (2015), are:

1. The uncertainty due to *natural variability in alongshore wave power* (*Py*) is originates in predictions from to the inability of the model to predict the (semi) random order of future wave events. Additionally input reduction, a method generally used to keep calculation times within workable limits (Walstra et al, 2013), inevitably goes along with loss of detail in temporal (and temporal?) resolution. Hence, natural variability is a source of uncertainty. To estimate this uncertainty 20 years of offshore observations of waves at measurement station Noordwijk (ca. 50 km from HBD, see Figure 2) are translated to points along the -8 m+NAP depth contour of the planned nourishment with SWAN (Simulating WAves Nearshore, Booij et al., 1999). The alongshore sediment transport gradient is determined for each time-step with the CERC formula (CERC, 1984). In this way the yearly average alongshore component of the wave power is related

to the sediment transport gradient. For the years in the dataset the mean alongshore wave power varied between -0.41 and 0.33 kW/m (negative value indicates waves from the south).

2. The uncertainty due to (*seasonal and spatial) variations in profile shape* is introduced by limitations of the model. Present day process based models are able to reproduce morphological changes during storm conditions but the performance is highly variable when modeling recovery of the foreshore and beach, introducing uncertainties in the instantaneous state of the cross shore profile, the associated alongshore transport and transport gradient. The relation between the profile steepness and the alongshore transport gradient is established with UNIBEST-TC (Bosboom, 1997; Bakker, 1995) for 40 years of measured natural profiles for a nearby beach (Castricum, see Figure 2). The measured RMS profile steepness at Castricum for these years varied between 1:17 and 1:43 and was on average 1:33.

3. The m*odel uncertainty* is assessed by doing a large number of sensitivity calculations with the 2DH process based morphological model with different model settings and schematization choices such as wave seeding, spin-up effects, beach heads and tidal asymmetry (Kroon et al., 2015). The model uncertainty includes contributions of uncertainty of other components in the model chain (i.e. SWAN, CERC, UNIBEST TC) however it does not claim to be complete. The tested range varied between a variation in volume losses with a factor 0.5 to 2.0.

4. The *calibration uncertainty* is based on an estimate of the uncertainty in the calibration data due to measurement errors and outliers. The tested range varied between a variation in volume losses with a factor 0.5 to 2.0.

Table 1. Summary of sources of uncertainty.

Uncertainty	Method to estimate range	Result	Data density
Natural variability in wave forcing	20 years of wave observations SWAN + CERC	Relation between yearly average alongshore wave power (P) and alongshore bulk volume loss (V)	20 calculations
Variations in profile shape	40 years of cross shore profile measurements at Castricum + UNIBEST	Relation between profile steepness (s) and alongshore bulk volume loss (V)	40 calculations
Model uncertainty	Sensitivity calculations with 2DH model	Probability density function of alongshore bulk volume loss (V)	5 * 3 calculations sampled into probability density function of (V)
Calibration uncertainty	Measurement uncertainty in calibration data translated to upper and lower limit of calibration parameters.	Probability density function of alongshore bulk volume loss (V)	3 calculations interpolated into probability density function of (V)

Table 1 gives an overview of the derived relations and the density of the dataset. In the present study the probability density functions, that relate input and model uncertainty to sediment transport gradients, are used as sample basis to expand the dataset to train the BN. Only one variable is varied that means that the input data set contains no correlation between parameters. This assumption significantly decreases the dimensionality of the problem but will reduce the accuracy of the uncertainty estimation. However, accuracy is not the major purpose of this paper, i.e. exploring the concept of BNs in morphological uncertainty assessments. The resulting BN is presented in Figure 3. The histograms show the probability distributions for each variable, based on the training dataset, and the arrows show the dependencies as defined in the network setup. Figure 3 shows the prior probability distributions, being solely based on the training dataset. When additional evidence for one or more variables is introduced, posterior probability distributions are presented. This will be shown in Section 4.1.
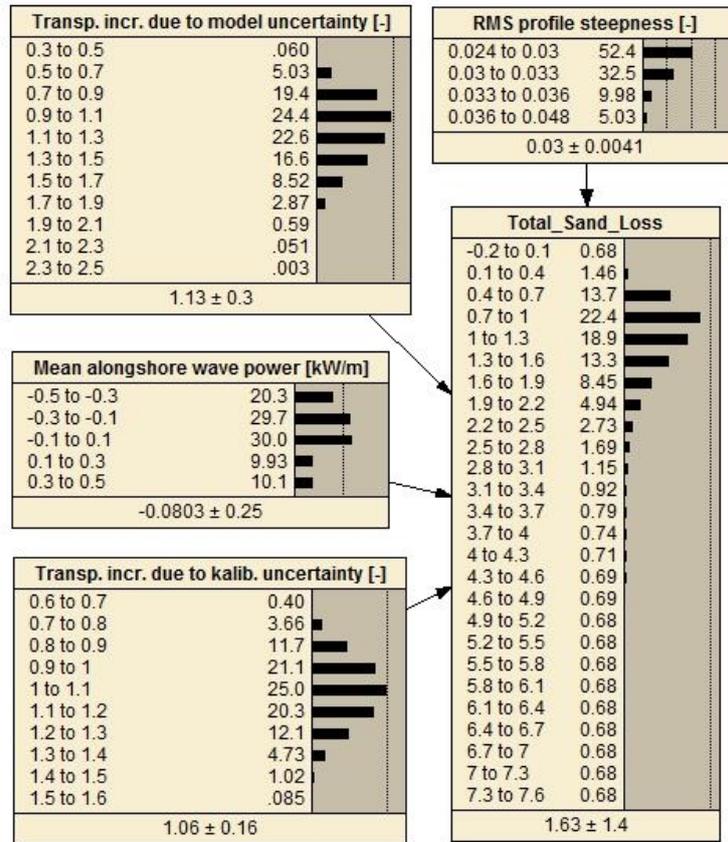
Figure 3: Bayesian Network for uncertainty propagation in process based morphological modeling.

2.5 Face validity

The aggregation of training data is inherently accompanied by some loss of accuracy. A common first test for Bayesian Networks is a face validity check. The network is said to be face valid if the response of the model is in agreement with the expectations of an expert and or the input data (Den Heijer, 2013; Pitchforth, 2013). For instance one would expect that the likelihood of a high bulk volume loss increases for the observation high wave energy.

Face validity is only a limited test of the skill of a network however in data poor environments it is often the one of the few methods available.

2.6 Bias, skill and Log-likelihood ratio

The prediction skill of the model can be quantified using linear regression, in the shape of $\hat{y} = \alpha + \beta\hat{x}$, to estimate the relation between predictions and observations. In which the regression parameters α and β are a measure for the bias of the prediction (Den Heijer, 2013). The skill is than determined as the normalized root mean square error of the regression ($\hat{y}_i$):

$$s = 1 - \frac{\sum_{i=1}^{I} \sigma_{x,i}^2 [y_i - \hat{y}_i]^2}{\sum_{i=1}^{I} \sigma_{x,i}^2 [y_i]^2} \tag{2}$$

In which the variance $\sigma_{x,i}^2 = \sum_{i=1}^{I} p(F_i|O_j)[D_j - \hat{x}_i]^2$ and the Bayesian mean value $\hat{x}_i = \sum_{i=1}^{I} p(F_i|O_j)D_j$.

The log-likelihood ratio is used to indicate the skill of the updated probability distributions by comparison of the posterior likelihood of an observation with the probability of the forecast of that observation. This is mathematically written as:

$$LLj = log[p(F_i|O_j)]_{F_i=O_j} - log[p(F_i)]_{F_i=O_j} \tag{3}$$

The first term on the right is the posterior probability of forecast F_i given observation O_j and the second term is the probability of forecast F_i. If the log-likelihood ratio exceeds 0 the posterior distribution is an improvement of the prediction. If the log-likelihood ratio is less than 0 this means the posterior likelihood is lower than the prior likelihood.

3.   Evidence

3.1   HBD observations

The constructed Bayesian Network trained with model data is compared to observed losses in the HBD project area (so called evidence). The evidence used to test the BN is derived from one year of cross shore profile observations at the HBD. The considered definition for the alongshore wave power, profile steepness and volume loss are described in the sections below. Figure 4 presents the evidence used for model testing and updating.
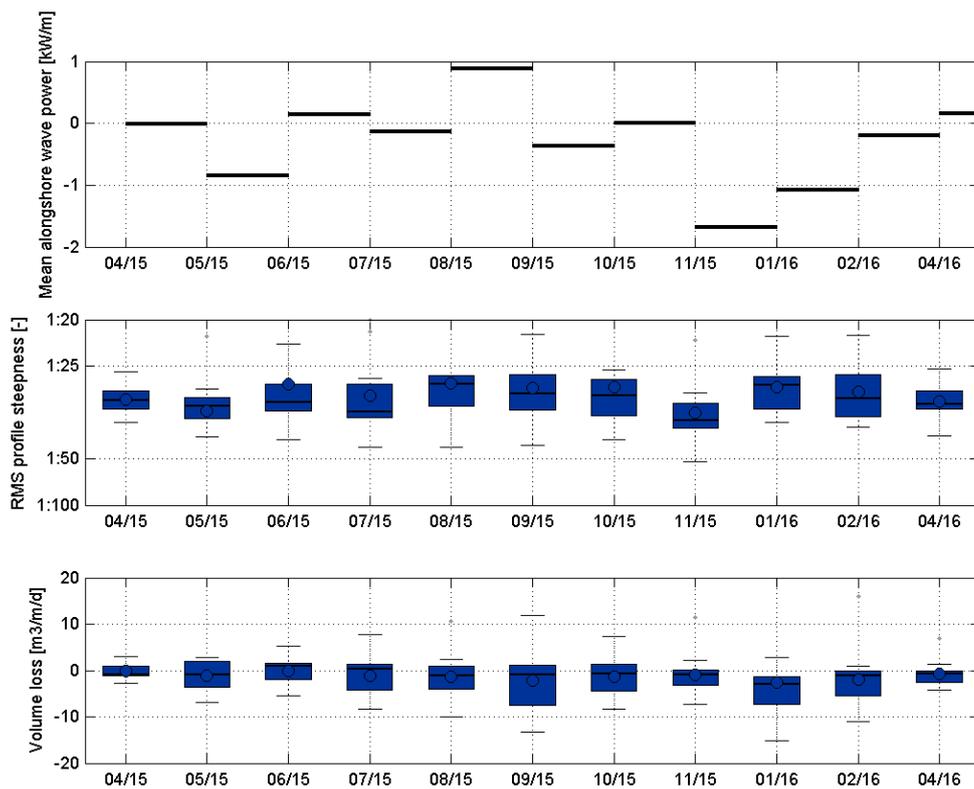

Figure 4: Observations of alongshore wave power (top), RMS profile steepness (middle) and volume loss (bottom) in the first year after construction of the HBD.

3.1.1   Alongshore wave power

To combine the effect of wave height, wave direction and wave power the observed wave height at measurement station IJmuiden Munitiestortplaats (MUN), Figure 2, are expressed as the directional alongshore wave power available for sediment transport (de Schipper, 2013).

The first year after construction is dominated by high waves from the southwest, increasing the long term average net transport potential approximately 2.5 times.

3.1.2   *Steepness observations*

The alongshore transport is dominated by wave energy and angle and profile steepness is only a second order effect according to Mil-Homens (2016). Nevertheless the same study shows that the alongshore transport has a very high correlation with the root mean square downward slope. The RMS slope is here defined as the seaward facing part of the slope on which wave breaking can occur (grey line in Figure 5), and is calculated as:

$$s_{rms} = \sqrt{\frac{\sum_{i=0}^{N} s_i^2}{N}} \tag{3}$$

Where s_i are the discrete slope values between each (uniform spaced) bathymetry point of the active profile. The RMS profile steepness accounts for local increase in profile steepness due to the presence of nearshore sandbars.
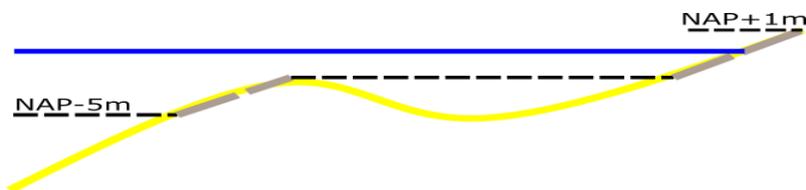

Figure 5: Definition of RMS profile steepness as the seaward facing part of the slope on which wave breaking can occur, indicated with the grey dashed line.

3.1.3   *Bulk volume loss*

The bulk volume loss for each transect is defined as the volume loss per meter of the active profile from -7 m+NAP to the dune foot at 3 m+NAP.
The observed net sediment loss in alongshore direction in the first year after construction of the HBD was approximately 3x higher than the local sediment gradient before nourishment.

3.2   Evidence selection

To make a fair test of the model performance several transects around the transition from erosive to accretive coast are discarded. At a certain location to the north and the south of the curved coastline of the HBD the eroding trend changes to an accretive trend. The model does not predict the location of this transition exact and as a consequence the sign of the predicted volume change can be opposed to that of the observation. The BN does not include any information on coastline orientation or gradient. Therefore it is felt that including transects close to these transition locations would penalize the model too strong.

4.   Results

4.1   Face validity

Once the Bayesian Network is constructed the first step is to check its face validity. With respect to wave power one would expect a valid model to display higher sediment losses in case of a high wave power, irrespective of the wave direction since we are dealing with a coastal outcrop. The upper panel of Figure 6 shows that in case of high waves with a Northwestern direction the peak of the probability density distribution is indeed increased from approximately 0.8 to 1.2 and that the Bayesian mean changes from 1.6 to 2.0.
On a gentle slope waves break over a wider region with less intensity reducing the total alongshore sediment transport compared to a steep slope in which the same amount of energy is dissipated over a much smaller zone. So a valid model would predict lower sediment losses for more gentle profiles. In the lower panel of Figure 6 the network is conditioned for observations of profile steepness. It shows that the likelihood of high volume loss is reduced in case of a gentle profile steepness of approximately 1:40. When compari

ng prior and posterior volume loss in Figure 6 it can be seen that the peak remains at the same location but the weight of the distribution is shifted from a Bayesian mean of approximately 1.6 to 1.4.
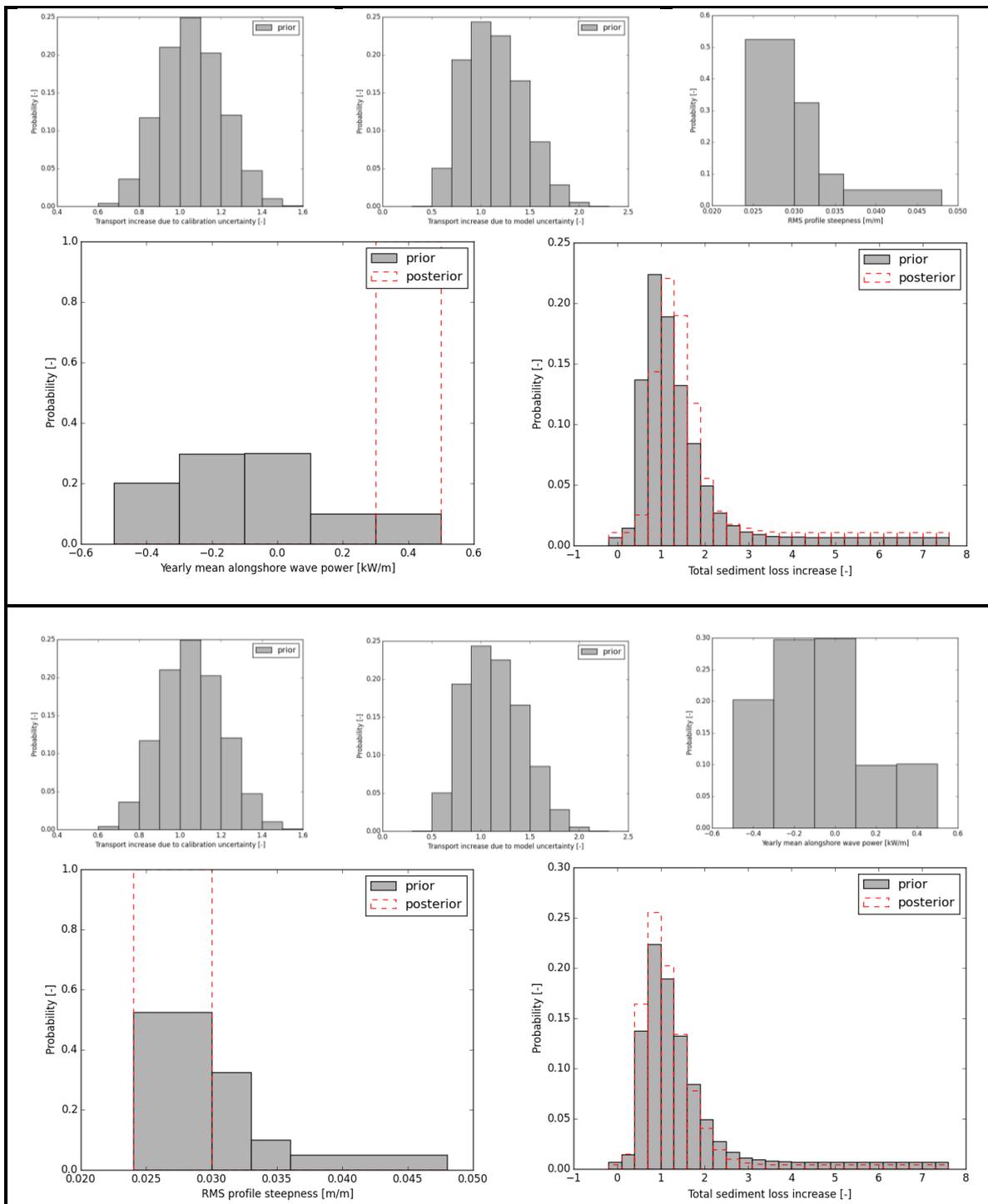

Figure 6: Face validity for gentle sloping profile (left) and high waves from the NW (right).

4.2   *Bias and skill*

The observations of the first year after construction of the HBD can be used to condition the network. The r

elatively high SW alongshore wave power, relatively steep slope can be the cause of the high observed relative volume loss. However, a high relative volume loss can also indicate a bias in the predicted volume losses.

The first year observations for the transects indicate a model bias with linear regression parameters α = 0.9 and β=1.7. This means that the BN underestimates the volume loss and that this underestimation increases for higher volume losses. The regression model skill is 0.9, this means that the error is relatively small compared to the regression estimate.

In addition to the bias and regression skill the log-likelihood is determined. The log-likelihood test the skill of the prediction simultaneously with the skill of the uncertainty prediction. The log-likelihood values are determined for the likelihood of the observations given the wave power observation, given the profile steepness observation and given both observations. The log-likelihood values are summarized in Table 2. The log-likelihood values are low due to the significant prediction bias. However, positive values indicate that the observations increase the probability of the observed bulk volume loss.

The log-likelihood given the profile steepness observation is in general much higher than the log-likelihood given the wave power observation. This means that the model expects a stronger increase in bulk volume loss due to steep profiles than due to increase in wave power. This is an interesting notion since data analysis shows that the correlation in the observations is much stronger between wave power and bulk volume loss than profile steepness and bulk volume loss.

Table 2. Log-likelihood values.

	$LL_{Wave\ power}$	$LL_{Steepness}$	$LL_{Wave\ power+Steepness}$
Observations averaged over transects	0.07	0.25	0.32
Transect 3	0.03	0.02	0.08
Transect 4	0.02	0.00	0.04
Transect 7	0.01	0.44	0.46
Transect 8	0.03	0.42	0.44
Transect 10	0.02	0.29	0.31
Transect 11	0.02	-0.01	0.04

4.3   Update Network

In Figure 7 the network is conditioned with the average of the observations in time and space (upper panels). The lower panels of Figure 7 show the posterior distribution of the free nodes model uncertainty and calibration uncertainty. The peak of both distributions is shifted right, indicating a model underestimation, and the width is increased, indicating less certainty. Decrease of certainty is caused by contradicting observations or, in this case, by observations with low prior probabilities. If observations would be very close to the most probable observations, the posterior distributions would become more narrow. In this case if all observations would fall within the bin with the highest probability the standard deviation of the relative bulk volume loss would decrease with 50% from 1.4 to 0.7.
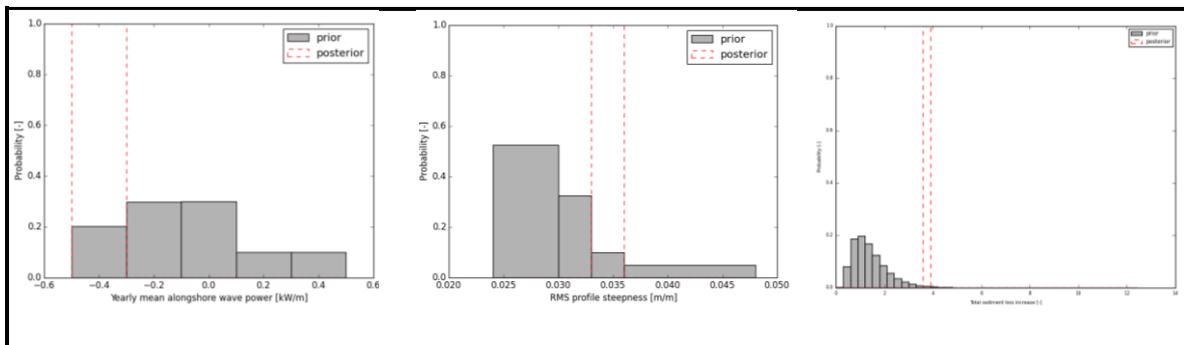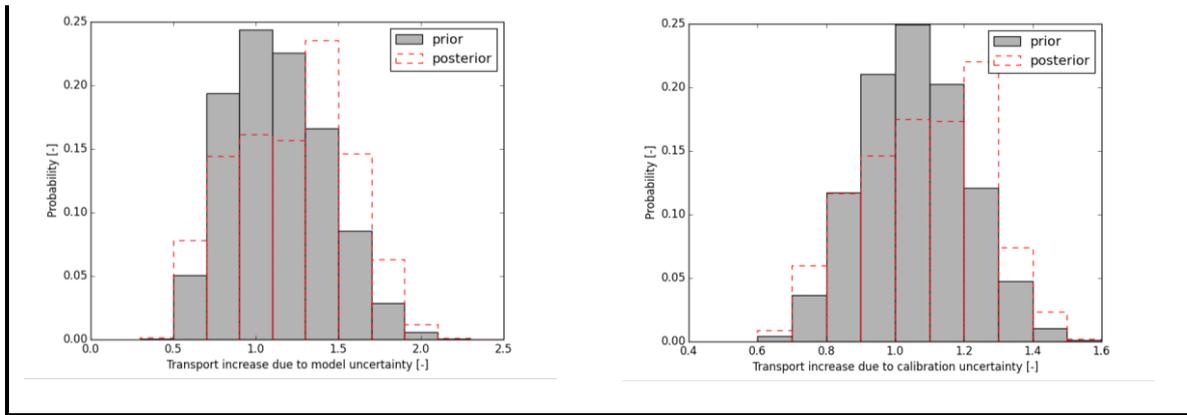
Figure 7: Update of network with first year of observed values (upper panels) and adjusted free nodes (lower panels).

5. Discussion

5.1 *Forecast update*

One of the possible applications of BNs in long term morphology is that of forecast updating. As the observed bulk sediment losses in the first year after construction appear to be significantly higher than the predicted sediment losses a logical question of a responsible authority or private party would be "can we expect higher losses than expected for future maintenance as well?".

The updated BN can serve to answer this question. In Figure 8 the posterior distribution of the sand loss is presented after adoption of the model and calibration probabilities given the observations of the first year (lower panel of Figure 7). Ideally the posterior distribution becomes more narrow indicating a higher certainty after updating of the network. In this case the opposite is true, the certainty has reduced after updating the network, most likely caused by the underlying assumptions of the model. The updating process basically tells us that we know less about the sediment loss mechanisms than we anticipated in advance (before the analysis of the collected data). Nevertheless, the shift of the peak of the distribution from approximately 1.2 to 2 does give an answer to the question posed. It is expected that future losses will be higher than the prior forecast as well, however care is advised in the interpretation of these results since the certainty has decreased.
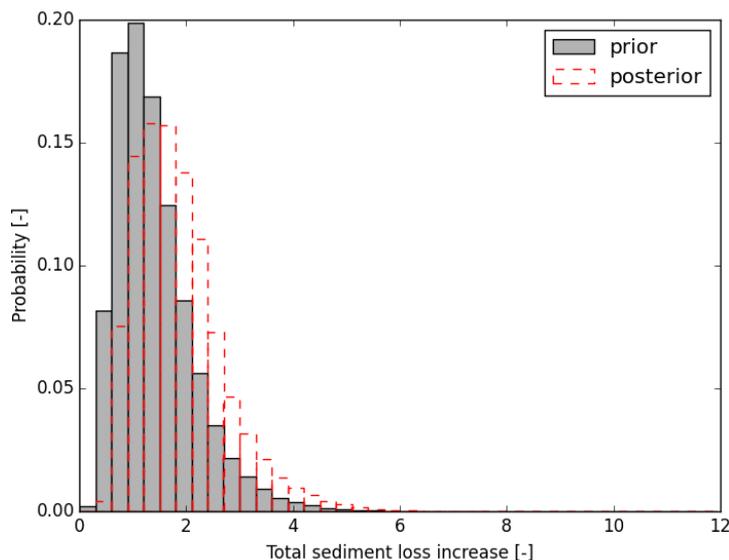

Figure 8: Prior and posterior probability density distribution for the relative bulk sediment loss after updating of model and calibration uncertainty in the BN.

5.2   *Full systemic translation of uncertainties*

Chapter 4 presents a successful transfer of uncertainty in model inputs and model instruments to a prediction of a coastal state indicator. The most important question that remains is how can the resulting uncertainty bandwidth be validated. Given that an observation is always only one realization of many possible observations.

The next step is to explore other concepts for probabilistic assessments such as amongst others ensemble modeling and Kalman filtering. If a BN approach seems most viable in comparison to these methods, further validation is required. For this a more complete model data set with less bias and which includes joint probabilities need be generated for model training. Additionally a larger validation data-set with more observations is essential.

6.   Conclusions

This paper investigates the use of Bayesian networks for a systemic approach of uncertainty assessments in long term morphological predictions on a decadal scale, as one of many possible concepts for probabilistic assessments. For this purpose a simple BN was set up that included uncertainties in wave forcing, profile steepness, model uncertainty and calibration uncertainty to an uncertainty bandwidth around the output parameter of choice; bulk sediment volume loss in alongshore direction. The BN is neither complete, in terms of sources of uncertainty, nor perfect, in terms of forecasting volume loss, but is suitable to explore the BN concept for morphological modelling purposes.

The use of a BN is inevitably accompanied by loss of detail. Nevertheless logical tests show the model responds as expected, and can be said to be *face valid.* Positive log-likelihood values tell us that the network exhibits a similar response to forcing and steepness as is contained in the field observations. That is, the likelihood of the observed bulk volume loss is increased by the observations of wave forcing and profile steepness. The model does have a significant bias, however, validation of the predictive value was not the purpose of the work presented in this paper. The BN approach allows for the use of evidence to update the model (uncertainty) and to adjust a forecast accordingly. Whether this adjusted forecast is an improvement was not tested here.

We have shown that even with a limited number of calculations a face valid BN that displays a similar response to field observations can be obtained. The BN has shown to have the potential to transfer uncertainties of different sources to the required coastal state indicator. At this stage it is not possible to make any statements on the quality of the forecast. For that purpose it is necessary to generate a training data set in which multiple sources of uncertainty are varied simultaneously. However, the major challenge remains the validation of the predicted uncertainty bandwidth.

The adjusted forecast for the life-time of the maintenance buffer helps the responsible contractors and government authorities with the planning of the future nourishments. Most importantly, we have shown that updating of the forecast is theoretically possible without additional computationally expensive hindcasts. That is, under the condition that the observations lie within the bounds of the training dataset since a BN cannot extrapolate.

Acknowledgements

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