MODELING OF COASTAL CURRENTS USING COUPLED OCEAN-WAVE MODEL CONSIDERING WAVE-CURRENT INTERACTION ON RANDOM WAVES

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Abstract

The Stokes drift velocity profile on random waves in ocean model is generally used by regular wave approximation as functions of significant wave height and another representative wave parameters. Estimation of the Stokes drift velocity should be considered based on full directional wave spectra in ocean model. However, the computation is much expensive since ocean model needs full directional spectra information passed from wave model for the calculation of Stokes drift velocity profile. This study proposes the coupled ocean-wave model considering directional spectra for calculation of the Stokes drift velocity profile and assumption of the two-dimensional Gaussian spectra as a simplified directional wave spectra. To validate the assumption and to verify the model performance, simulations of Stokes drift velocity is carried out for an ideal simple bathymetry including one slit at the center to mimic coastal area. Finally, coastal current hindcast is performed for Tanabe Bay of Wakayama Prefecture in Japan. The Stokes drift velocity due to random waves gives weaker on mean velocity in comparison with the regular wave approximation.

Key words: Stokes drift, coupled ocean-wave model, random waves, directional wave spectrum

1. Introduction

The numerical ocean model needs to consider various physical mechanisms using parameterization for understanding ocean circulation system. One of important physical mechanism is the wave-induced transport, well known as Stokes drift. It plays an important role for the upper ocean current system and has been a subject of recent researches. However, many studies about the Stokes drift on random waves have been limited to considering regular waves using significant wave height or another representative wave statistics to simplify the modeling, especially in depth-limited regions. Breivik et al. (2014) proposed an approximation of the parameterized Stokes drift velocity profile considering random wave effects but it focused on deep water region.

In numerical ocean model the vortex force and the Coriolis-Stokes force can be dynamically considered as wave conservative effect in the term of wave effects on currents (hereafter called WEC) in Eulerian wave-averaged current equations for mass and momentum. The Stokes drift effect is included in the vortex force. The vortex force formalism of the Regional Ocean Modeling System (ROMS; Shchepetkin and McWilliams, 2005) was applied to the surf zone by Uchiyama et al. (2010), also including the application of current effect on waves (CEW). The application of WEC and CEW enables us to investigate the effects of wave-current interaction on random waves in shallow water by accurately estimating Stokes drift velocity profile to be inserted to the vortex force term. However, computing the Stokes drift profile is expensive because it involves evaluating integral with the two-dimensional wave spectrum at every desired vertical level, and additional also at every mesh in simulation of coastal current.

This study aims the parameterized Stokes drift for the depth-limited region using a coupled ocean-wave model. It can consider the wave-current interaction on random waves by incorporating the Stokes drift velocity profile calculated by full directional spectra, which can be computed in Simulating WAves Nearshore (SWAN). We propose an approximation of directional spectra by the two-dimensional Gaussian distribution. It can reduce number of variables from full directional spectra to four and can reduce message passing costs greatly. The random wave characteristics represent only two parameters as frequency dispersion and directional speed based on the two-dimensional Gaussian distribution approximation. Although the assumption neglects detail shape of directional spectra and multi wave system conditions, they are not significant in shallow water region.

First, we examine simple ideal test for the Stokes drift velocity profile using the two-dimensional

Gaussian distribution approximation. Second, two types of numerical simulations are conducted for validation and application. One is simple ideal bathymetry with a slit at center of the bathymetry giving regular wave information (referred to as wave1d) and random waves considering parameterized wave spectra (referred to as wave2d). Another simulation is carried out for coastal current analysis targeting Tanabe Bay of Wakayama Prefecture in Japan. The impacts of directional spectra in the Stokes drift components and mean velocity are discussed by two numerical simulations in detail.

2. Methodology

2.1 Governing Equation

A system equation for hydrostatic current system considering waves on current can be described as follow (McWilliams et al., 2004). The wave induced acceleration term, following J included in the right hand side of the equation is expanded and applied to the surf zone (e.g. Uchiyama et al., 2010).

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla_{\!\!\perp})\mathbf{u} + \mathbf{w}\frac{\partial \mathbf{u}}{\partial z} + \mathbf{f}\hat{\mathbf{z}} \times \mathbf{u} + \nabla_{\!\!\perp} \boldsymbol{\varphi} - \mathbf{F} = -\nabla_{\!\!\perp} \mathbf{H} + \mathbf{J} + \mathbf{F}^{\mathbf{w}}$$
(1)

where $\mathbf{u} = (u(x, y, z), v(x, y, z))$ is the Eulerian velocity in horizontal direction, *t* is time, *z* is the vertical coordinate, \hat{z} is the vertical unit vector, *f* is the Coriolis parameter, ϕ is the momentum balance, *F* is the non-wave non-conservation forces, F^w is the wave non-conservation forces, *H* is Bernoulli head, and ∇_{\perp} is the horizontal differential operator. The wave induced acceleration term **J** is expressed by vortex force (McWilliams et al., 2004; Uchiyama et al., 2009),

$$\boldsymbol{J} = -\boldsymbol{g} \times \frac{1}{\partial \zeta^c + h} \boldsymbol{u}^{St} \boldsymbol{\nabla} \times \boldsymbol{u}$$
⁽²⁾

where g the gravity acceleration, u^{St} the horizontal Stokes drift velocity, $\zeta^c = \zeta + \hat{\zeta}$ is the corrected phased averaged surface elevation by the wave effects on currents. The u^{St} can be given by the Stokes velocity based on the wave model.

2.2 Formulation of the Stokes Drift on Random Waves

The Stokes drift velocity for random waves can be described (Kenyon et al., 1969)

$$u^{st}(z) = -2g\int E(\mathbf{k}) \frac{\mathbf{k}}{\omega(k)} \frac{k\cosh 2k(z+D)}{\sinh 2kD} d\mathbf{k}$$
(3)

$$w^{St}(z) = -\boldsymbol{\nabla}_h \cdot \int \boldsymbol{u}^{st} dz \tag{4}$$

where $E(\mathbf{k})$ the wave number spectra and D the total water depth considering wave averaged mean water change, $D = h + \zeta + \zeta^c$, w^{St} the vertical Stokes drift velocity component and ∇_h is the operator for horizontal partial differentiation. Eq. (3) can be simplified for random waves in deep-water

$$u^{st}(z) = \frac{16\pi^3}{g} \int E(\mathbf{k}) \frac{\mathbf{k}\omega^3}{k} e^{2kz} d\mathbf{k}.$$
 (5)

Breivik et al. (2014) discussed the influences of high-frequency contribution to velocity profile for deepwater waves. However, these components will be quickly decreased but low-frequency contribution is expected to significant influences for intermediate to shallow water waves. Moreover, it is difficult to analytically parameterize Eq. (3) by given directional spectra in shallow water waves due to hyperbolic functions in Eq. (3).

Although the effects of random waves are important, use of full spectra information is too expensive for communication in the coupling model. Therefore, we use minimum information of directional spectra based on frequency and directional spreading from the spectra. We mainly focus on spectrum shape near the peak. The directional spectra can be assumed by Two-dimensional Gaussian spectrum (Mori et al., 2011).

$$E(\omega, \theta) = \frac{m_0}{2\pi\sigma_\omega\sigma_\theta} \left\{ -\frac{1}{2} \left[\left(\frac{\omega - \omega_0}{\sigma_\omega} \right) + \left(\frac{\theta - \theta_0}{\sigma_\theta} \right) \right] \right\}$$
(6)

where ω and θ are wave frequency and direction, ω_0 and θ_0 are the principal frequency and direction, σ_{ω} and σ_{θ} are the standard deviation of frequency and direction, and m_0 is the variance of the surface elevation. The directional spread s of Eq. (6) at the principal ω_0 can be obtained analytically,

$$s = \delta_{\theta} = \sqrt{\int d\theta \theta^2 D(\theta)}$$
(7)

$$= \frac{1}{\sqrt{2\pi\sigma_{\theta}}} \int \theta^{2} exp \left[-\frac{1}{2} \frac{(\theta - \theta_{0})^{2}}{\sigma_{\theta}^{2}} \right] d\theta$$
(8)
$$= \sigma_{\theta}$$
(9)

where $D(\theta)$ is directional distribution which is normalized to one, $\int D(\theta)d\theta = 1$. Therefore, σ_{θ} in Eq. (6) is equal to the standard directional spread and is measured in radians.

In general, the directional distribution of wave spectra depends on both direction and frequency but here we make the simplifying assumption that it is independent on frequency. This assumption is valid around the peak frequency where most of the wave components is focused on. For the Mitsuyasu-type $\cos^{2S} \theta/2$ directional distribution, the directional spread can be expressed in terms of the power of the cosine function,

$$\sigma_{\theta} \simeq \sqrt{\frac{2}{1+S}}$$
(10)
$$D(\theta) = D_0 \cos^{2S} \frac{\theta - \theta_0}{2}$$
(11)

where $D(\theta)$ is the Mitsuyasu-type directional distribution. Therefore, σ_{θ} , directional spread *s* and *S* of the $\cos^{2S}\theta/2$ distribution can be interchanged. The observed directional spread by buoy at peak frequency is roughly about 10 degrees (0.17 rad) for swell and is 30 degrees (0.52 rad) for wind sea. Thus, the directional spread σ_{θ} ranges from 0.15 to 0.5 depending on the sea states. For the frequency dispersion, the width of frequency spectra can be calculated by

- 1	(12)
$Q_p = \frac{1}{\sqrt{\pi}\sigma_\omega}$	
$\sqrt{n}o_{\omega}$	

Furthermore, there is a relation between Q_p and the Joint North Sea Wave Project (JONSWAP) spectral shape parameter γ as $Q_p = -0.015\gamma^2 + 0.06\gamma + 1.37$.

The approximation of two-dimensional Gaussian spectra is less accurate in higher frequency region (i.e. Breivik et al., 2014). However, it is expected that the high frequency components are rapidly decreased vertically. As the target is intermediate to shallow water region, therefore, the saturation of high frequency components is not obtained as wind wave equilibrium spectra f^{-4} in deep-water. We try to express full wave spectra by two parameters directional and frequency bandwidths (S and Q_p) here after.

2.3 Numerical model

The Coupled Ocean-Atmosphere-Wave-Sedimentation Transport (COAWST) Modeling System developed by Warner et al. (2010) is utilized for coupling models between the Regional Ocean Modeling System (ROMS) and Simulating WAves Nearshore (SWAN) to exchange data fields. Wave spectra are resolved by SWAN and considered in vortex force term on WEC of ROMS to resolve Stokes drift velocity profile. At that time to apply the computational costs reduction the directional spectra in SWAN are converted with the two parameters and approximately expressed as the two-dimensional Gaussian distribution. Note that, as mentioned above, the approximation targets shallow water and is valid around the peak frequency where most of the wave components is focused on and based on.

3. Model Validation

In this section two types of theoretical comparison are conducted as validation of the approximated formulation of Stokes drift by the two-dimensional Gaussian spectra. One aims to validate the theoretical Stokes drift velocity profile derived from the two-dimensional Gaussian spectra through the two parameters, directional spreading and frequency. This test aims how consideration of directional spectra for the velocity profile of Stokes drift with and without directional spectrum information. The model test giving regular wave information is referred to as wave1d and the test giving random wave information is referred to as wave2d hereafter. Another aims to perform Stokes drift simulations for both cases, wave2d and wave1d, on an ideal simple constant slope bathymetry with an inlet for wave diffraction and shoring effects. This simulation examines the difference of Stokes drift velocity behavior between wave2d and wave1d in the magnitude and the direction from offshore to onshore through the slit.

3.1. Profiles under Approximated Spectra

The full directional wave spectra are calculated in spectrum wave model and are considered as approximated directional spectra through the two parameters in calculation of Stokes drift velocity profile in ocean circulation model. Before simulating coupled wave and ocean model, this paragraph shows the theoretical profile of Stokes drift velocity based on the approximation of directional spectra at depths of 10m, 50m and 100 m. In this case wave height is assumed as 0.5 m, frequency is 10.0s and the effects of directional and frequency on the Stokes velocity are shown below.

Figure 1(a) shows the profiles of Stokes drift velocity at depth of 100 m ($Q_p = 2.0, S = 7.0$). The Stokes drift velocity on the surface differs slightly from one another. The profiles considering only frequency dispersion (referred to as wave1d-freq: green line) in Figure 1(a) and considering only directional spread (wave1d-dir: blue line) in Figure 1(a) are basically smaller than the profile of regular wave (referred to as wave1d: black line) in Figure 1(a). Therefore, the profile considering directional spectrum (referred to as wave2d: red line) in Figure 1(a) is smaller than the others. This seems due to wave energy dissipation concerning consideration of nonlinear wave. The vertical profiles' trend at depth of 50m in Figure 2a ($Q_p = 2.0, S = 7.0$) is similar to that trend and the profile of wave2d case much smaller than the other profiles relatively with in Figure 1(a) in the upper half of water column. Figure 3(a) which is the profiles of Stokes drift velocity at depth of 10m ($Q_p = 1.0, S = 7.0$) explicitly shows the difference of

Stokes drift velocity one another. Following Figure 3(a), vertical integrated velocity of the Stokes drift in waveld-freq and waveld-dir is almost same but the profiles are quite different one another. The profiles' gradient in case of wave2d, waveld-dir and wave2d gradually increases with upward and is similar. On the other hand, the profile gradient in case of wave1d-freq drastically increases with upward and the horizontal components of Stokes drift velocity in case of wave1d-freq take higher in upper surface than in case of wave1d. Frequency dispersion-depend contribution prominently decreases with being close to bottom.

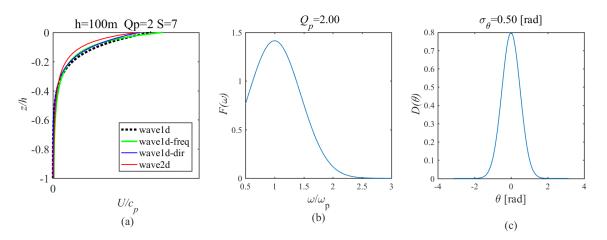


Figure 1. (a) The Stokes drift velocity profile for the full directional spectrum (red line), random wave with frequency dispersion (green line), random wave with directional spread (blue line) and under the monochromatic directional spectrum (black line) for the case of depth h=100 m, Goda's spectral bandwidth Q_p =2.0 and Mitsuyasu-type directional distribution parameter S=7.0. (b) The frequency spectra. (c) The directional spectra.

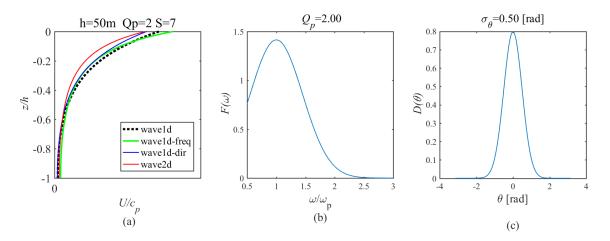


Figure 2. (a) The Stokes drift velocity profile for the full directional spectrum (red line), random wave with frequency dispersion (green line), random wave with directional spread (blue line) and under the monochromatic directional spectrum (black line) for the case of depth h=50 m, Goda's spectral bandwidth Q_p =2.0 and Mitsuyasu-type directional distribution parameter *S*=7.0. (b) The frequency spectra. (c) The directional spectra.

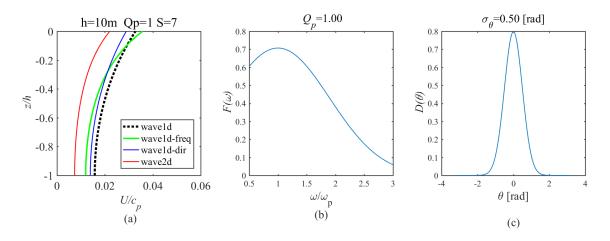


Figure 3. (a) The Stokes drift velocity profile for the full directional spectrum (red line), random wave with frequency dispersion (green line), random wave with directional spread (blue line) and under the monochromatic directional spectrum (black line) for the case of depth h=10 m, Goda's spectral bandwidth Q_p =1.0 and Mitsuyasu-type directional distribution parameter *S*=7.0. (b) The frequency spectra. (c) The directional spectra.

3.2. Test Simulations on an Ideal Simple Bathymetry

The test simulation to evaluate Stokes drift effect with directional spectrum information is carried out by the numerical model. We simulate the characteristics of Stokes drift on an ideal simple bathymetry with constant slope (Inlet test, see Figure 4). A uniform and mild slope (1/640) with a small aperture in the center of domain are set up and bathymetry becomes uniform at the depth of 4 m in nearshore. Domain size is square of 15.4 km east-west and 14.4 km north-south with 200 m spatial resolution, and 20 vertical layers. The offshore and lateral boundaries are absorb boundary condition but the onshore is closed boundary condition, respectively. The incident wave condition is 1.0 m in significant wave height propagating from offshore to onshore. The simulations were performed for wave2d and wave1d cases, respectively. In wave2d case, two different frequency dispersions were given by changing JONSWAP parameter, $\gamma = 3.3$, 10 and three different directional dispersion $\sigma_{\theta} = 10$, 50, 90 were given. Totally six cases were examined for sensitivity analysis of Stokes drift effect on random waves.

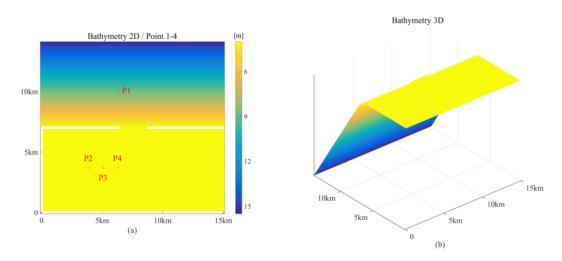


Figure 4. Bathymetry of idealized simulation (so-called the inlet test) with constant slope (1/640) with a slit in the center. (a) Top view. (b) Bird view

The results of wave2d case show that the spatial spreading of 1 hour averaged wave induced current on the surface are more widely spread behind of the slit than wave1d. Then, the magnitude of one hour averaged Stokes drift velocity in wave2d case is smaller than wave1d due to linear combination of directionality. The Stokes drift velocity in regular wave is typically overestimated due to directional spreading. In the case of $\gamma = 3.3$, $\sigma_{\theta} = 10$ (see Figure 6), the narrower directional spreading condition gives concentrated wave induced currents along the wave direction behind the slit than in the wider directional spreading condition as shown in Figure 5. In the case of $\gamma = 3.3$, $\sigma_{\theta} = 10$, namely wider frequency dispersion with narrower directional spreading, the magnitude of Stokes drift is decreased in lower layer more remarkably than in upper layer in wave2d in comparison with wave1d case as shown in Figure 5 and 6. The results shows good agreement with the theoretical validation in section 3.1. The Stokes drift velocity profile drastically decreases in downward in case of wave1d-freq than in case of wave1d-dir (c.f. Section 3.1.).

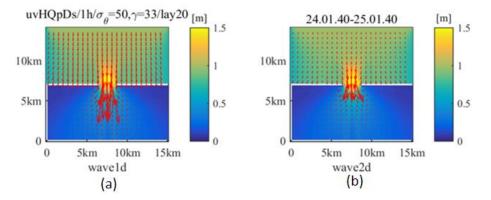


Figure 5. Spatial distributions of one hour mean velocity at layer of 20/20 (vectors: the Stokes drift velocity, contour: significant wave height). The offshore wave conditions are $\gamma = 3.3$, $\sigma_{\theta} = 50$. (a) Waveld. (b) Wave2d.

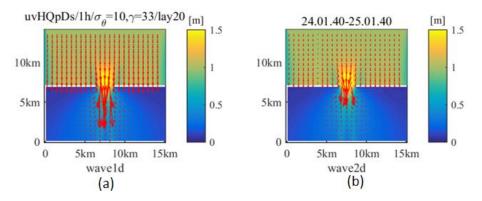


Figure 6. Spatial distributions of one hour mean velocity at layer of 20/20 (vectors: the Stokes drift velocity, contour: significant wave height). The offshore wave conditions are $\gamma = 3.3$, $\sigma_{\theta} = 10$. (a) Waveld. (b) Wave2d.

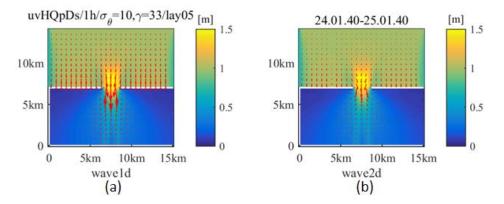


Figure 7. Spatial distributions of one hour mean velocity at layer of 5/20 (vectors: the Stokes drift velocity, contour: significant wave height). The offshore wave conditions are $\gamma = 3.3$, $\sigma_{\theta} = 10$. (a) Waveld. (b) Wave2d.

Figure 8 shows time series of Stokes drift velocity on the surface at fix points of P1 to P4 (see Figure 4) for the case of $\gamma = 3.3$, $\sigma_{\theta} = 50$. The solid line and dotted line in Figure 8 indicate the results of wave2d and wave1d case, respectively. The time series of Stokes drift velocity in wave1d case consistently show slower velocity at the any points than in wave2d case due to wave frequency and directional dispersion effects. The magnitude of Stokes drift at P4 is largest, followed in order by P3 to P2 since the effect of diffract easily appears at the point far from the slit.

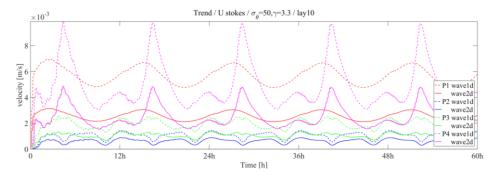


Figure 8. Time series of 1 hour mean velocity at fixed locations at the layer of 20/20 (solid line: wave2d, d otted line: wave1d). The offshore wave conditions are $\gamma = 3.3$, $\sigma_{\theta} = 50$.

4. Hindcast for Tanabe Bay of Wakayama Prefecture

A series of simulations was carried out for Tanabe Bay in Wakayama Prefecture in Japan (see Figure 9). Tanabe Bay faces the Pacific Ocean and the analysis of coastal current using the coupled ocean-wave model was carried out and validated as an application to real situation. As similar to the inlet test in section 3.2, the numerical simulations were performed both wave2d case and wave1d case. In addition, uncoupled run was carried out for without wave effects (referred to as ROMS run) for comparison to investigate the impact of wave-current interaction own.

The domain of the simulations is 30.0 km east-west and 30.0 km north-south with 500 m spatial resolution and 10 vertical layers. The simulations were integrated over the period 30 September to 10 October 2009 excluding 7 days spin-up time. There was typhoon approach at the end of period. The initial and boundary conditions were given by the operational mesoscale spectrum model grid point value (MSM-GPV) dataset from Japan Meteorological Agency for meteorological forcing. It doesn't include long wave radiation and then it was implemented with Automated Meteorological Data Acquisition System (AMeDAS) data. JCOPE2 data and TPXO7.2 data are used as three dimensional velocity and astronomical tide for lateral boundary in ROMS, respectively. Sommerfeld boundary condition is setup as free surface propagation at the lateral boundaries.

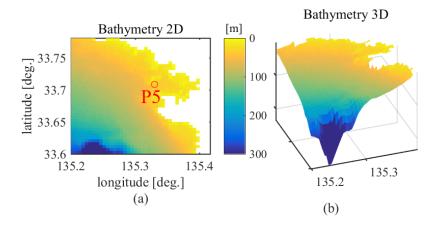


Figure 9. Bathymetry of Tanabe Bay simulation. (a) Top view (b) Bird view.

4.1 Spatial Distribution of Coastal Current

Figure 10 shows snapshot of spatial distribution of mean velocity in the cases of wave2d and wave1d at 18:40 in 5 October 2009. We explored in section 3 the effects of considering directional spectra in calculation of Stokes drift velocity profile. The velocity ranges on the order of mean velocity to the power minus two; nevertheless as can be seen in Figure 10 it is found the difference of mean velocity in not only the magnitude but also the direction. Furthermore, the mean velocity near the coastal area is reproduced in wave2d case as smaller than in wave1d case.

The application of vortex force formalism has been extended to the surf zone in terms of WEC and also CEW. Thus, there is a possibility of evaluation of wave fields to be affected by the change of how Stokes drift velocity profile is expressed in ocean model. Actually, as can be seen in Figure 10 the current effect on waves appears in the distribution of significant wave height. Significant wave height is higher in wave2d on the whole than in wave1d. The trend is remarkable especially in the west part of the simulated domain.

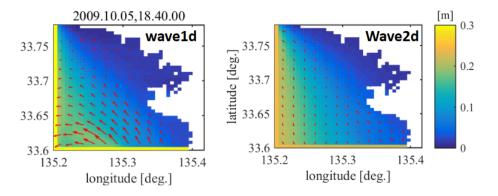


Figure 10. The snapshot of mean velocity for the simulation on Tanabe Bay of Wakayama Prefecture. The vectors indicate mean velocity. Shade is significant wave height. Upper most layer (20/20). (Left) Wave1d. (Right) Wave2d.

4.2 Time Series of Mean Velocity

For comparison among wave2d case, wave1d case and ROMS run in mean velocity, Figure 11 shows mean velocity time series at P5 (c.f. Figure 9) in each case in upper figure and the ratio of one of wave2d case divided wave1d case in lower figure. There is 30 % mean velocity difference between wave2d and wave1d. The results of ROMS run tends to be entirely smaller than one of the others. It is found that simulations of these cases differ one another also in time variation.

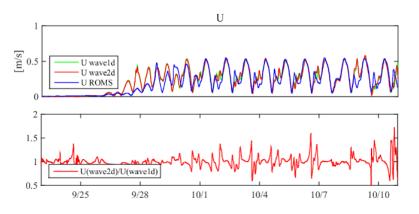


Figure 11. The time series results of simulation on Tanabe Bay of Wakayama Prefecture. Red, green and blue line are wave2d case, wave1d case and ROMS run, respectively in upper figure. Red line of lower figure is ratio of one of wave2d case divided wave1d case.

5. Conclusions

The description of wave directional spectra proposed here for calculation of Stokes drift velocity profile has been shown and used for coastal current simulations. It focuses on spectra shape near the peak to include minimum information of directional spectra. Then, we can derive directional spectra from the two parameters of frequency dispersion and directional spread by the two-dimensional Gaussian spectra assumed.

First we validated the assumption for profile and horizontal components in theoretical comparison. It was indicated that frequency dispersion contributed well only in near the surface in theoretical verification and that was explicitly verified also in simulations of Inlet test.

Utilizing the novel assumption in passing two parameters between ocean and wave model to reduce the computational costs, namely full directional spectra can be derived from them, typical overestimation of Stokes drift velocity was improved with the coupled ocean-wave model. The assumption can be targeted to shallow water since it is expected that the high frequency components are rapidly decreased vertically, therefore, we could examine simulations for Tanabe Bay of Wakayama Prefecture for coastal current analysis. Important contribution of consideration of full directional spectra in calculation for Stokes drift velocity is that accurate Stokes drift velocity calculation reflects the impact also mean velocity in the magnitude and the direction.

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