Coastal Dynamics 2017
Paper No. 186

ON THE MORPHODYNAMIC EVOLUTION OF MEGA-NOURISHMENTS

Antonios Valsamidis[1], Dominic Reeve[2], Yuzhi Cai[3] and Nicholas Dodd[4]

Abstract

In this study, a modified version of the one-line model is presented, which includes an advection term. This term corresponds to the impact of a longshore current on shoreline evolution. The new equation may describe the morphodynamic evolution of mega-nourishments which extend far enough to the offshore distance to be exposed to sea currents. New solutions are presented corresponding to two different initial conditions: a. a rectangular mega-nourishment; and b. a bell-curved mega-nourishment. In both cases, the longshore current causes a migration of the mega-nourishment in the same sense as the current. Further, as the littoral transport rate increases so the erosion rate of the mega-nourishment increases with associated feeding of the adjoining beaches. The initial shape of the mega-nourishment influences to a great degree the morphodynamic evolution of the beach.

Key words: one-line model, shoreline evolution, mega-nourishment, analytical and semi-analytical modelling, beach model, coastal defense scheme

1. Introduction

In the present day, almost half of the world population lives in coastal areas. However, 70% of the beaches worldwide (Dean and Dalrymple, 2004) are subjected to erosion. Moreover, the continuous population and industrial growth which have been taking place near coastal areas for many decades in combination with the climate change and the consequent sea level rise (Nicholls et al, 2007), put in danger the coastal habitants and ecosystems (Villatoro et al., 2014). To deal with these problems, coastal engineers have applied the following two different kinds of coastal defence methods, with primary aim to maintain the shoreline front: a. hard coastal defence schemes; and b. soft coastal defense schemes. Hard beach protection techniques include groynes, breakwaters, revetments, seawalls and other similar structures. When installed, they change permanently the natural environment, plus, they can be very costly. However, they are necessary when a beach is exposed to violent hydrodynamic conditions. On the other hand, soft beach protection techniques are more sustainable with the environment and less costly. A typical such soft beach protection technique is beach nourishment, (e.g. Dean, 2002; Ojeda et al., 2008; Castelle et al., 2009; Yates et al., 2009; Kuang et al., 2011; Roberts and Wang, 2012; Cooke et al. 2012; Burcharth et al., 2015; Luo et al., 2015), in other words, the periodical feed of a beach with sediment material to mitigate the adverse impacts of erosion. Over time, the typical volume of beach nourishments has been increased from $100\ m^3/m$ of distributed sediment material in the cross-shore direction before the 1970s (de Schipper, 2016) to 400 – 600 m^3/m (van Duin et al., 2004; Ojeda et al., 2008) in more recent times. In all cases, the effectiveness of the beach nourishment applications depends on the maintenance of the deposited sediment material in the area where it was originally placed. The inevitable loss of sediment material due to the local hydrodynamic forcing is compensated with re-nourishment of the beach from time to time. However, concerns have been raised over the latter practice as this may have negative impacts on the fauna of the nourished site (e.g. Peterson and Bishop, 2005; Speybroeck et al., 2006; Janssen et al., 2008).

Aiming at a more efficient soft coastal protection method, mega-nourishments as a new strategy of coastal defence has been introduced (Stive et al., 2013). The fundamental concept of this technique is

[1]College of Engineering, Swansea University, Bay Campus, SA1 8EN, Swansea, Wales, UK. antonios.valsamidis@swansea.ac.uk
[2]College of Engineering, Swansea University, Bay Campus, SA1 8EN, Swansea, Wales, UK. d.e.reeve@swansea.ac.uk
[3] School of Business and Management, Swansea University, Bay Campus, SA1 8EN, Swansea, Wales, UK. y.cai@swansea.ac.uk
[4] Faculty of Engineering, University of Nottingham, University Park, NG7 2RD, Nottingham, UK nicholas.dodd@nottingham.ac.uk

to apply a massive amount of marine aggregates to a coastal site and allow the dominant hydrodynamic forces to physically redistribute the sediment material along the beach providing in this way a long-term beach protection at regional scale. As the forces of nature are expected to complete a major portion of any mega-nourishment project, the latter is characterized by relatively low cost. An adverse impact of mega-nourishments is that in the short-term they might obstruct the longshore sediment transport (Brown et al., 2016).

The first application of a mega-nourishment occurred on the sandy western coast of the Netherlands, between the cities of Rotterdam and The Hague, and near the North Sea basin (de Schipper et al., 2014). The latter is not very deep with water depth ranging between *20* and *80m*. In this area, *21.5* million cubic metres of marine aggregates were transported and consequently deposited for the formation of an artificial peninsula which extends *3 km* along the beach and *1 km* maximum distance in the offshore-ward direction (de Schipper et al., 2014), (Figure 1). This project was named the Sand Engine, (Zandmotor, originally in Dutch). The first years following the construction of the Sand Engine have been characterized by its beneficial impact (in terms of nourishing material) on the surrounding beaches (de Schipper et al., 2014; de Schipper et al., 2016). Generally, though, the impacts of mega-nourishments can be both positive (for instance increasing the width of adjusted beaches) and negative (disrupting the longshore sediment transport), thus, sufficient modelling and monitoring work is required for supplying the decision makers with adequate information before any intervention to the beaches is planned (Capobianco et al., 2002). Moreover, the worldwide-observed trend for an increase in size and volume of nourishments, in conjunction with their sophisticated geometric design, enhances the need for sufficient predictions of their morphodynamic evolution in time (Luijendijk et al., 2017).

Figure 1. The evolution of the Sand Engine between a. July 2011; and b. October 2012 (adopted from de Schipper et al., 2014).

To this end, process-based models which can simulate the morphodynamics of mega-nourishments have been recently applied. Luijendijk et al. (2017) used Delft 3D for the execution of a structured model experiment where it was shown that primary factors for the erosion of the Sand Engine is the wave climate and the vertical tide accounting for 75% and 17% of the total eroded volume, respectively. The impact of horizontal tide, winds and surges was found to be rather of secondary importance.

Arriaga et al. (2017) presented and applied a computational model named Q2D-morfo for the simulation of the long-term morphodynamic evolution of Sand Engine. This model incorporates the littoral drift caused by breaking waves, and assesses in a simplified way cross-shore sediment transport without considering tides. Results showed a diffusive behaviour of the shoreline, and a *30-year* prediction of the morphodynamic evolution of the Sand Engine demonstrated a retreat of its amplitude from *960 m* to *350 m*. In addition, the adjoining shores on the left and right-hand side of Sand Engine advanced *80* and *100 m* towards the sea, along a *2.5 km* beach section, respectively. Finally, Brown et al. (2016) investigated hypothetical applications of mega-nourishments via computational methods in the UK. However, as Luijendijk et al. (2017) note, nor the degree of accurate predictions via the state-of-the-art models neither the dominant factors of the morphodynamic evolution of mega-nourishments have adequately been studied yet.

Thus, the current work aims at further investigating the phenomenon of mega-nourishments via analytical modelling techniques, and specifically, analytical solutions to the one-line model (e.g.

Pelnard-Considère, 1956; Reeve 2006; Walton and Dean, 2011). A main advantage of this modelling strategy is that it can be utilized for the segregation and subsequently, the independent study of specific dominant coastal processes, without having to consider the full complexity of a coastal system (Zacharioudaki and Reeve, 2008). Moreover, analytical solutions to the one-line model are ideal for conceptual design purposes, as their evaluation is relatively easy and does not exhibit numerical instability (Valsamidis and Reeve, 2016). Therefore, they can be used for validating the outputs of numerical models for simplified cases before proceeding to the full simulation of a given coastal site via numerical modelling techniques (Hanson 1987). The new version of the one-line model that is going to be presented includes, except for the diffusion term, an additional advection term. The latter corresponds to a shoreline-orientation dependent source that is incorporated as feedback to the utilized hydrodynamic model and vice versa. This enhanced version of the one-line model is consequently applied for the investigation of the morphodynamic evolution of some simple cases of mega-nourishments.

Subsequently, this study is organized as follows: In Section 2 the new version of the one-line model is developed which describes the shoreline evolution due to the presence of a longshore current near the beach, in addition to the littoral drift caused by breaking waves. Then the new partial differential equation that is derived is solved via analytical means and two analytical solutions are yielded corresponding to mega-nourishments with different shapes. In Section 3, the new analytical solutions are evaluated and results are presented. A discussion on the new analytical solutions and the results is provided in Section 4, and finally, conclusions on this study are illustrated in Section 5.

2. Methodology

A relatively simple modelling approach was adopted, aiming at visualizing the basic elements of morphological evolution of a mega-nourishment. Specifically, a traditional one-line modelling technique which is applicable to cases where the shoreline position y, with reference to a datum line, is a single-valued function of the longshore distance x. In other words, no recurves in plan shape are considered in the modelling procedure. Thus, the method deployed in this study is not directly applicable to the Sand Engine since the latter was designed as a hook-shaped nourishment (Figure 1a) rather than a rectangular or bell-shaped one.

Initially, the conservation of mass is applied to the equilibrium profile. Apart from waves breaking obliquely to the shore, a longshore current with velocity V is assumed. We take this current to be parallel to the x-axis. At any point along the shoreline, it will have local cross-shore and longshore components, respectively. This current is not fully parallel to the local shoreline gradient. Moreover, is hypothesized that the longshore current transfers sediment material, and the corresponding component of sediment flow towards the beach is denoted as Q_1 (Figure 2). The direction of Q_1 is identical to the direction of the V_y vector.

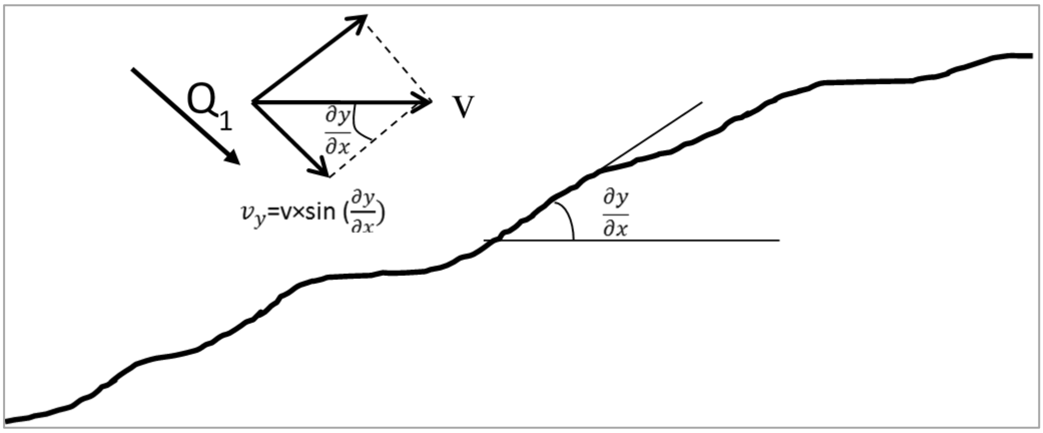

Figure 2. A longshore current with velocity V. Its cross-shore component with velocity Vy produces a sediment flow towards the beach: Q_1.

Under these conditions, applying conservation of mass to the equilibrium profile yields:

$$\Delta x\Delta y(D_B+D_C)+\frac{\partial Q}{\partial x}\Delta x\Delta t+Q_1\Delta t=0 \Rightarrow \Delta x\Delta y(D_B+D_C)+\frac{\partial Q}{\partial x}\Delta x\Delta t+V\sin\left(\frac{\partial y}{\partial x}\right)\Delta x(D_B+D_C)\Delta t=0$$
$$\Rightarrow \Delta y(D_B+D_C)+\frac{\partial Q}{\partial x}\Delta t+V\sin\left(\frac{\partial y}{\partial x}\right)(D_B+D_C)\Delta t=0 \quad (1)$$

where D_B is the berm height; D_C is the depth of closure; and Q is the parallel to the local gradient, sediment flow.

Considering $\left(\frac{\partial y}{\partial x}\right) \ll 1$ then $\sin\left(\frac{\partial y}{\partial x}\right)=\frac{\partial y}{\partial x}$ (2)

$$\text{Thus: } \Delta y(D_B+D_C)+\frac{\partial Q}{\partial x}\Delta t+V\frac{\partial y}{\partial x}(D_B+D_C)\Delta t=0 \Rightarrow \frac{dy}{dt}(D_B+D_C)+\frac{\partial Q}{\partial x}+V\frac{\partial y}{\partial x}(D_B+D_C)=0 \Rightarrow$$
$$\frac{dy}{dt}+\frac{1}{(D_B+D_C)}\frac{\partial Q}{\partial x}+V\frac{\partial y}{\partial x}=0 \quad (3)$$

However, $Q=Q_0(2a_o-2\left(\frac{\partial y}{\partial x}\right))$, (Larson et al., 1987) (4)

where a_o is the direction of breaking waves with respect to an axis parallel to the shoreline orientation (Figure 3); and in addition, the wave angle a_o is constant.
Substituting Equation 4 into Equation 3 leads to the following extended version of the one-line model:

$$\frac{dy}{dt}=\varepsilon\frac{\partial^2 y}{\partial x^2}-V\frac{\partial y}{\partial x}=0 \quad (5)$$

where ε is a diffusion coefficient which characterizes the rate of shoreline evolution. More information about the diffusion coefficient ε and how is assessed can be found in the report by Larson et al. (1987). Equation 5 includes an advection term: $V\frac{\partial y}{\partial x}$ which describes the impact of a longshore current on shoreline evolution.

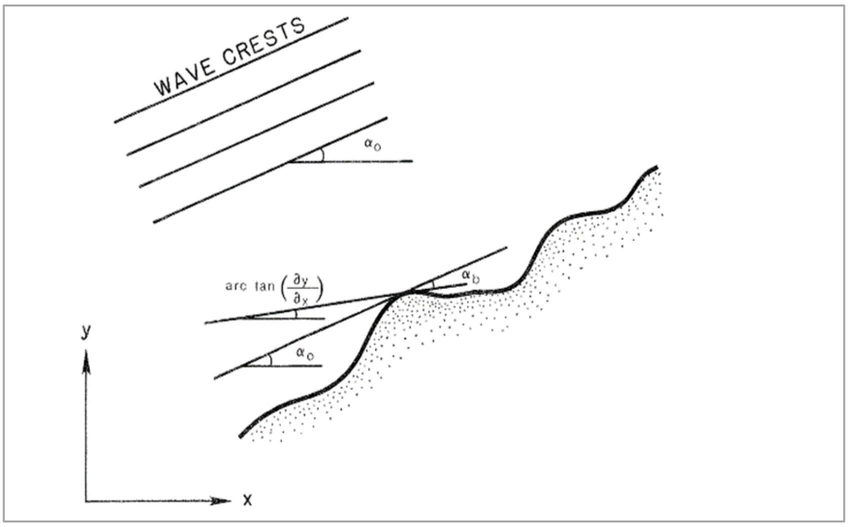

Figure 3. Wave angles with respect to the orientation of a beach (adopted from Larson et al., 1987).

The advection term in Equation 5 can be physically explained as follows: the sea current erodes the updrift side of the mega-nourishment and then deposits the eroded material on its downdrift side. In this way, the mega-nourishment is assumed to slide in the direction of the current. Specifically, on its up-drift side (Figure 4): $\frac{\partial y}{\partial x}>0$ thus, $-V\frac{\partial y}{\partial x}<0$ which implies erosion. Similarly, on its down-drift side: $-V\frac{\partial y}{\partial x}>0$ which implies an accretive trend.

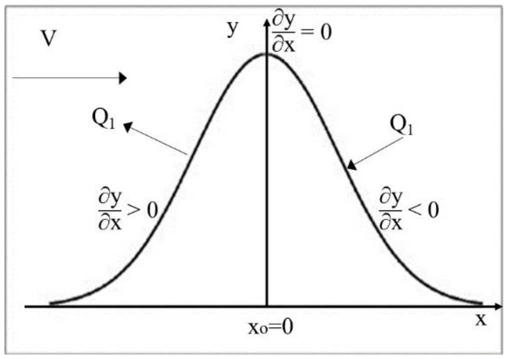

Figure 4. The longshore current with velocity V causes erosion on the updrift side of the mega-nourishment, and a symmetric accretion on its downdrift side.

2.1. ***Analytical solution for the case of a rectangular mega-nourishment***

Supposing as initial condition the existence of a rectangular mega-nourishment in the negative half-plane: $-\infty < x < 0$ with width *W* (Figure 5). Assuming a longshore current with velocity *V*, Equation 5 will be solved for the initial condition illustrated in Figure 5:

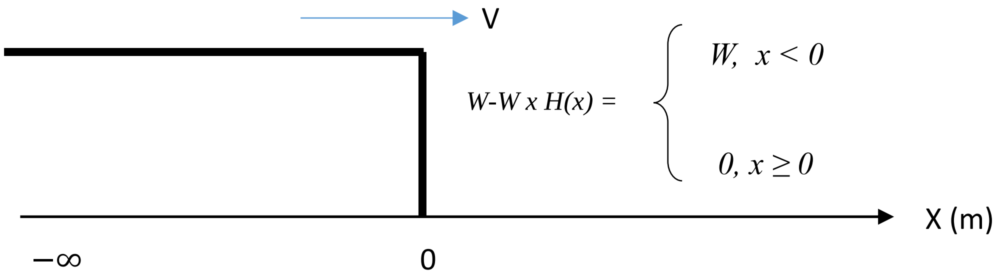

Figure 5. A rectangular mega-nourishment in the negative half-plane at *t=0* (adopted from Farlow, 1992).

We follow the development of Walton and Chiu (1979) and Farlow (1992) to mathematically express the problem as follows:

Governing equation: $\frac{dy}{dt} = \varepsilon \frac{\partial^2 y}{\partial x^2} - V \frac{\partial y}{\partial x}$ (6)

Subject to the initial condition: $y(x,0) = W - W\,x\,H(x)$ (7), over the domain: $-\infty < x < +\infty$

This problem may be solved with a change of variables:

$$\xi = x - Vt \quad (8)$$
$$\tau = t \quad (9)$$

Equations (6) and (7) are then transformed with respect to the new variables:

$$\frac{\partial y}{\partial \tau} = D \frac{\partial^2 y}{\partial^2 \xi} \quad (10)$$

with initial condition: $y(\xi,0) = W - W\,x\,H(\xi)$

(11)

The solution to this problem can be taken from tables of Fourier Transform Pairs, and is:

$$y(\xi,t)=\frac{1}{2\sqrt{\varepsilon\pi\tau}}\int_{-\infty}^{+\infty}\varphi(\omega)e^{-\frac{(\xi-\omega)^2}{4\varepsilon\tau}}d\omega,\ \text{where } \varphi(\omega) \text{ is the initial condition.}$$

Thus: $$y(\xi,\tau)=\frac{W}{2\sqrt{\varepsilon\pi\tau}}\int_{-\infty}^{0}e^{-\frac{(\xi-\omega)^2}{4\varepsilon\tau}}d\omega \tag{12}$$

Letting $\overline{\omega}=\frac{\xi-\omega}{2\sqrt{\varepsilon\tau}}$, it follows that $d\overline{\omega}=\frac{-1}{2\sqrt{\varepsilon\tau}}d\omega$, and so:

$$y(\xi,\tau)=\frac{W}{2}\left[\frac{2}{\sqrt{\pi}}\int_{\frac{\xi}{2\sqrt{\varepsilon\tau}}}^{+\infty}e^{-\overline{\omega}^2}d\overline{\omega}\right]\overset{\text{if } \xi<0}{\Longrightarrow} y(\xi,\tau)=\frac{W}{2}\left[\frac{2}{\sqrt{\pi}}\int_{\frac{\xi}{2\sqrt{\varepsilon\tau}}}^{0}e^{-\overline{\omega}^2}d\overline{\omega}+\frac{2}{\sqrt{\pi}}\int_{0}^{+\infty}e^{-\overline{\omega}^2}d\overline{\omega}\right]$$

$$\Rightarrow y(\xi,\tau)=\frac{W}{2}\left[-\text{erf}(\frac{\xi}{2\sqrt{\varepsilon\tau}}+1\right]=\frac{W}{2}\text{erfc}(\frac{\xi}{2\sqrt{\varepsilon\tau}}) \tag{13}$$

However, if $\xi \geq 0$, then:

$$y(\xi,\tau)=\frac{W}{2}\left[\frac{2}{\sqrt{\pi}}\int_{\frac{\xi}{2\sqrt{\varepsilon\tau}}}^{+\infty}e^{-\overline{\omega}^2}d\overline{\omega}\right]\Rightarrow y(\xi,\tau)=\frac{W}{2}\left[\frac{2}{\sqrt{\pi}}\int_{0}^{+\infty}e^{-\overline{\omega}^2}d\overline{\omega}-\frac{2}{\sqrt{\pi}}\int_{0}^{\frac{\xi}{2\sqrt{\varepsilon\tau}}}e^{-\overline{\omega}^2}d\overline{\omega}\right]$$

$$\Rightarrow y(\xi,\tau)=\frac{W}{2}\left[1-\text{erf}(\frac{\xi}{2\sqrt{\varepsilon\tau}})\right]=\frac{W}{2}\text{erfc}(\frac{\xi}{2\sqrt{\varepsilon\tau}}) \tag{14}$$

Therefore, for any real value of ξ, $y(\xi,\tau)=\frac{W}{2}\text{erfc}(\frac{\xi}{2\sqrt{\varepsilon\tau}})$, and if we set $\xi=x-Vt$ and $\tau=t$ we retrieve the solution in terms of the original variables as:

$$y(x,t)=\frac{W}{2}\text{erfc}(\frac{x-Vt}{2\sqrt{\varepsilon t}}) \tag{15}$$

2.2. *Semi-analytical solution for a mega-nourishment having a bell curve shape*

This time as initial condition was considered a shoreline shaped like a Gaussian curve. The mega-nourishment is exposed to wave action and a longshore sea current (Figure 6). Under these conditions, the shoreline evolution is described by the following system of equations:

Governing equation: $$\frac{dy}{dt}=\varepsilon\frac{\partial^2 y}{\partial x^2}-V\frac{\partial y}{\partial x} \quad -\infty < x < +\infty\ ,\ 0 < t < +\infty \tag{16}$$

Initial condition: $$y(x,0)=\varphi(x,0)=Be^{-\left(\frac{x-x_0}{a}\right)^2},\ -\infty < x < +\infty \tag{17}$$

Choosing $x_o=0$, the centre of the peak of the Gaussian curve will lie on the y axis, and $\varphi(x,0)=Be^{-\left(\frac{x}{a}\right)^2}$. Moreover, moving coordinates are considered to simplify the PDE by neglecting the $V\frac{\partial y}{\partial x}$ term, specifically, we set: $\xi = x - Vt$, and $\tau = t$.

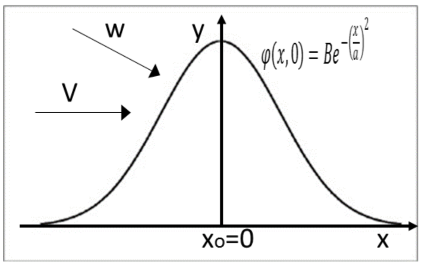

Figure 6. Schematic representation of a bell-curved mega-nourishment at *t=0*. It is exposed to a longshore sea current with velocity *V* and a constant wave event *w*.

Thus, the new problem to be solved is:

Governing equation: $\frac{dy}{d\tau} = \varepsilon \frac{\partial^2 y}{\partial \xi^2}$ $\quad -\infty < \xi < +\infty\,,\ 0 < \tau < +\infty$ (18)

Initial condition: $y(\xi,0) = Be^{-\left(\frac{\xi}{\alpha}\right)^2}$ $\quad -\infty < \xi < +\infty$ (19)

This problem can be solved via Fourier Transforms. Specifically, denoting the Fourier transform by F(.), we may write:

$$F\left(\frac{\partial y}{\partial \tau}\right)=\varepsilon F\left(\frac{\partial^2 y}{\partial \xi^2}\right) \Rightarrow \frac{d\bar{y}(\tau)}{d\tau}=-\varepsilon\omega^2\bar{y}(\tau) \tag{20}$$

$$F(y(\xi,0))=F\left(Be^{-\left(\frac{\xi}{a}\right)^2}\right) \Rightarrow F(y(\xi,0))=\frac{B\alpha}{\sqrt{2}}e^{\left(\frac{a\omega}{2}\right)^2}$$, according to Fourier Transform tables.

Thus, the solution is: $\bar{y}(\tau)=\frac{Ba}{\sqrt{2}}e^{\left(\frac{\omega\alpha}{2}\right)^2}e^{-\varepsilon\omega^2\tau}$ (21)

Next, the inverse transform to Equation 23 is taken:

$$y(\xi,\tau)=F^{-1}\left[\frac{Ba}{\sqrt{2}}e^{\left(\frac{\omega\alpha}{2}\right)^2}e^{-\varepsilon\omega^2\tau}\right] \Rightarrow y(\xi,\tau)=Be^{-\left(\frac{\xi}{\alpha}\right)^2} * \frac{1}{2\sqrt{\varepsilon\pi\tau}}e^{-\frac{\xi^2}{4\varepsilon\tau}} \Rightarrow y(\xi,\tau)=\frac{1}{2\sqrt{\varepsilon\pi\tau}}\int_{-\infty}^{+\infty}\varphi(\omega)e^{-\left(\frac{\xi-\omega}{4\varepsilon\tau}\right)^2}d\omega \tag{22}$$

Moreover, by returning to the original variables, $x=\xi-Vt$ and $t=\tau$ we find:

$$y(x,t)=\frac{1}{2\sqrt{\varepsilon\pi t}}\int_{-\infty}^{+\infty}\varphi(\omega)e^{-\left(\frac{x-V(t)-\omega}{4\varepsilon\tau}\right)^2}d\omega \tag{23}$$

Equation 23 cannot be evaluated analytically for all but the simplest forms of the incorporated integral. In contrast, for the great majority of cases, a numerical integration will be required for the assessment of the involved integral in the equation. For this reason, Equation 23 is regarded as a semi-analytical solution of Equation 5.

3. Results

3.1. *Rectangular-shaped mega-nourishment*

The evaluation of Equation 15 was executed for a rectangular mega-nourishment which extends *1000 m* in the offshore direction. Two different comparisons were made, the first one considering consecutive values of a constant longshore current velocity: *V= 0*; *0.1*; *0.5*; and *1 m/hr* for a constant wave event having height: *H=0.5m* and: period *T=6sec* (Figure 7). The second comparison was made for constant current velocity: *V=0.5 m/hr*, and a different value of wave height for each run of the model: specifically, *H=0*; *0.5*; *1*; *and 1.5 m/hr*, for the same wave period *T=6 sec* (Figure 8). In all cases, the analytical solution simulated 10 years of shoreline evolution. Results follow:

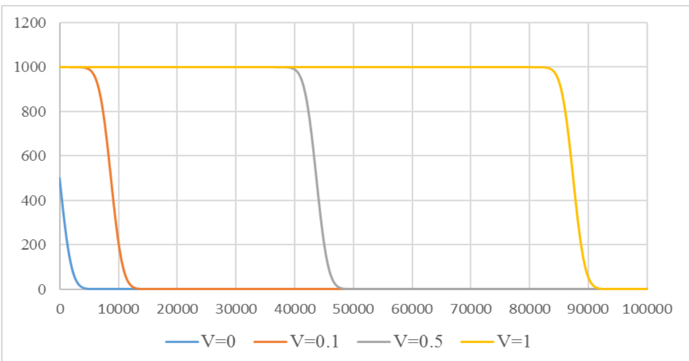

Figure 7. The ten-year morphodynamic evolution of the mega-nourishment for different values of the velocity of the longshore current: *V=0*; *0.1*; *0.5*; and *1* m/hr. and constant wave event (*H=0.5 m* and *T=6 sec*).

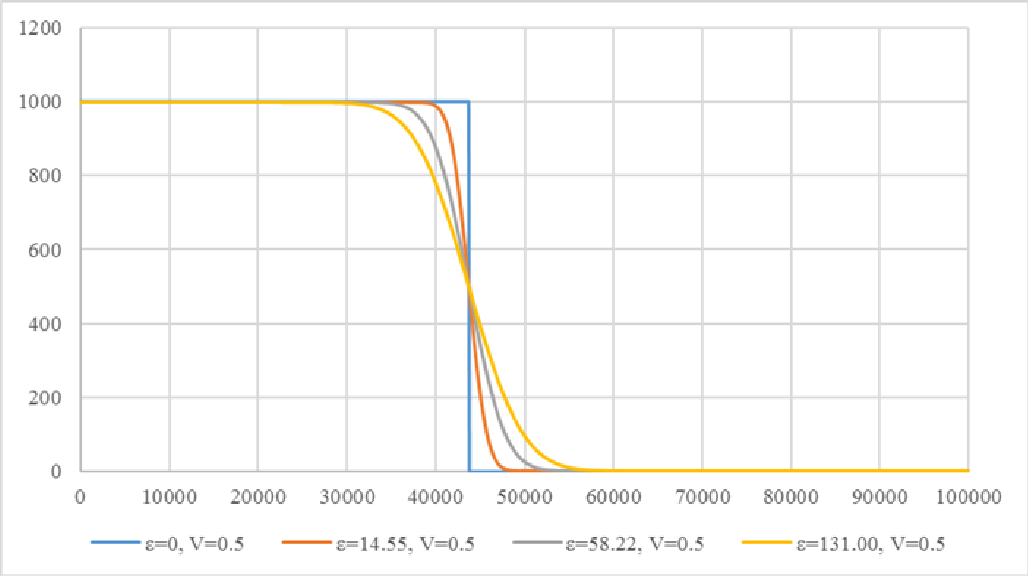

Figure 8. The 10-year morphodynamic evolution of a mega-nourishment for different values of the diffusion coefficient *ε*, corresponding to wave height *H=0; 0.5m; 1m;* and *1.5m*, and constant wave period *T=6 sec*. The longshore current is considered constant in time with *V=0.5 m/hr*.

3.2. *Bell-curved mega-nourishment*

The evaluation of Equation 23 illustrates the evolution in time of a bell-curved mega-nourishment. In analogy to Section 3.1, two different comparisons were made, the first one for constant current velocity: *V=0.05 m/hr*, and different value of the wave height in each run of the model: specifically, *H=0.5*; *1*; *and 1.5 m/hr*, for the same wave period *T=6 sec*. (Figure 10). The second one regarding successive values of the longshore current velocity: *V= 0*; *0.1*; *0.5*; and *1 m/hr* for a constant wave event having height: *H=0.5m* and period: *T=6sec* (Figure 11). In all cases, the semi-analytical solution simulated 10 consecutive years of uninterrupted shoreline evolution. Results follow:

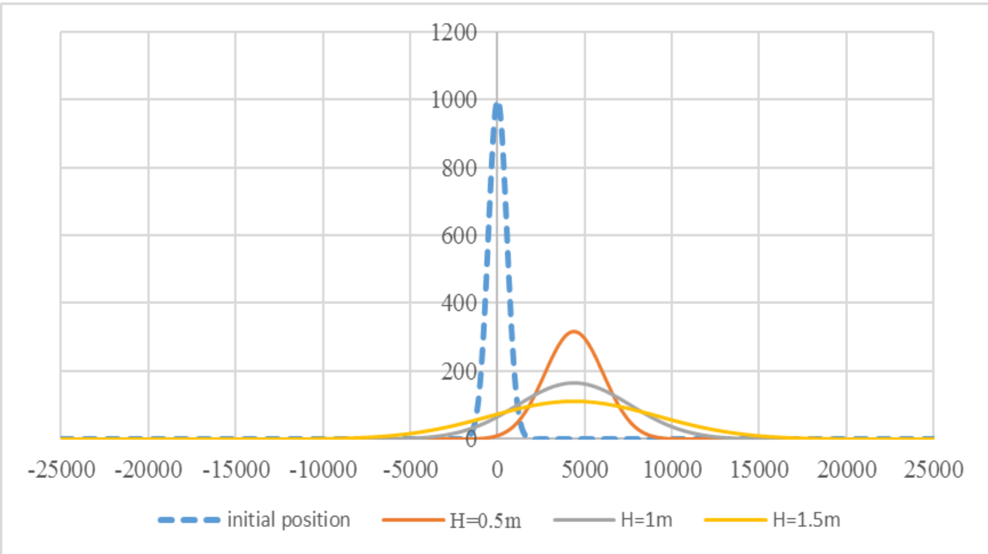

Figure 9. The 10-year morphodynamic evolution of a mega-nourishment for different values of the diffusion coefficient *ε*, corresponding to wave height *H=0*; *0.5m*; *1m*; *1.5m*. The velocity of the longshore current remains constant and equal to *0.5 m/hr*.

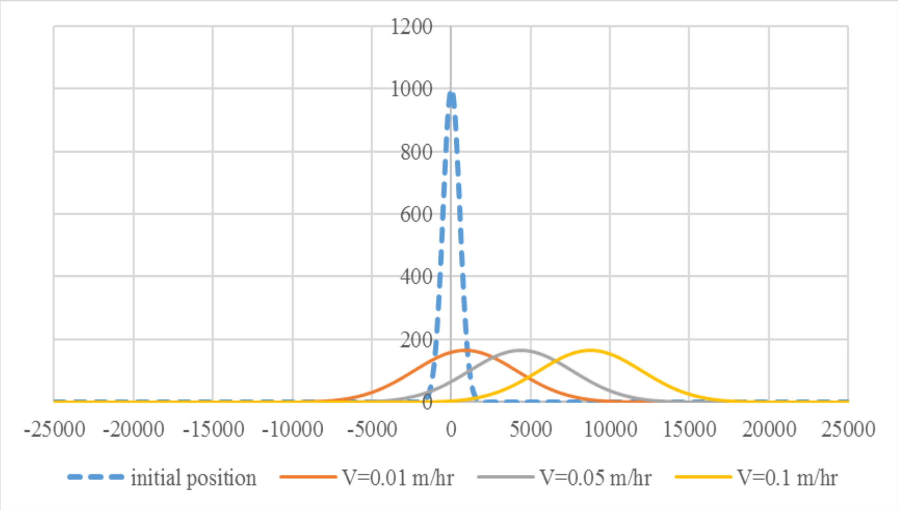

Figure 10. The 10-year morphodynamic evolution of a mega-nourishment for different values of the current's velocity *V=0.01*; *0.05*; *and*; *0.1*, and constant value of wave height *H= 0.5m*.

4. Discussion

Equation 5 is a new version of the one-line model assuming small shoreline gradients and constant wave forcing. The advection term appearing in this equation corresponds to a longshore current of constant velocity. Thus, Equation 5 could be useful for coastal areas in the world where longshore currents other than the littoral drift exist, for instance, tidal currents. Moreover, this equation was developed with the purpose of describing the morphodynamic evolution of mega-nourishments. And although its applicability may be broader, longshore currents might influence nourishments which extend further offshore rather than the smaller ones which are constructed near the natural beach.

The analytical solution to Equation 5 for two different initial conditions corresponding to different initial shapes of mega-nourishments (see Sections 3.1 and 3.2) led to Equations 15 and 23 which describe the shoreline evolution of a rectangular and bell-curved mega-nourishment, respectively. Equation 15 can be evaluated directly so it is an analytical solution, while for the evaluation of Equation 23, the integral involved must be assessed numerically. Therefore, this sort of solution to the one-line model is regarded a semi-analytical one (e.g. Reeve, 2006; Zacharioudaki and Reeve, 2008). For this work, the trapezoidal rule was utilized (e.g. Atkinson, 1988) for the assessment of the integral incorporated into Equation 23. A drawback of analytical and sometimes of the semi-analytical solutions is that they can be applied only for constant wave forcing and rather simplified case-studies. However, properly modified analytical solutions via a Heaviside scheme, which can incorporate time-varying wave-data have been recently produced (Walton and Dean, 2011; Valsamidis et al., 2013; Valsamidis and Reeve, 2016) for the case of shoreline evolution near a groyne. In addition, semi-analytical solutions with the indigenous ability to incorporate time-varying wave-data have been developed by Reeve (2006) and Zacharioudaki and Reeve (2008). Similarly, the analytical and semi-analytical solution presented in this study are being extended by the authors to incorporate time-varying input-data.

The evaluation of Equation 15 demonstrated that the initially rectangularly shaped mega-nourishment (see Section 3.1) feeds the downdrift side (the positive half-plane in Figure 5) due to the diffusive action of the wave forcing, plus the sea current which transports even further away the sediment material. Sea current velocities between *0* and *1 m/hr* were successively applied (Figure 7) for a constant wave incident. Moreover, from Figure 8 it is obvious that as the diffusivity gradually rises due to the corresponding rise in the value of wave height, the gradient of the propagating front of the mega-nourishment becomes milder.

As far as the bell curved mega-nourishment is concerned, Figure 6 suggests that as the wave forcing increases, the volume loss of the mega-nourishment increases as well, and is deposited over the adjoining beaches. In addition, the mega-nourishment migrates in the direction of the current. The latter phenomenon is more obvious in Figure 10 where different velocities of the sea current have been tested for a constant wave forcing.

5. Conclusions

The principal idea of mega-nourishments is to deposit a huge amount of sediment material in a specific place in a beach and then leave the forces of nature, namely the wave forcing and currents to redistribute the sediment material along the beach. In this way, long-term feeding of a beach with sediment material is ensured in regional scale. Due to the fact, though that this is a new technology, specifically, the first mega-nourishment was constructed in the Netherlands in 2011, the interactions of a mega-nourishment with its surrounding environment, and consequently, its positive and adverse consequences on the protection of coastal areas have not been studied adequately yet. This work aims to expand the predicting capability of existing process-based models for mega-nourishments, introduced a new version of the one-line model including an advection term (Equation 5). The latter describes the impact of a longshore current on a mega-nourishment. Subsequently, this new equation was solved analytically and an analytical and semi-analytical solution was derived. The first one describes the shoreline evolution on the downdrift side of a rectangular mega-nourishment. The second one refers to a bell-curved mega-nourishment. In both cases the evolution of the shoreline was assessed considering the combined forcing of waves and the longshore current. However, the new solutions cannot incorporate time-varying input-data. Thus, in a future work they will be modified properly to gain the ability to do so. In addition, the new solutions will have to be validated versus field-data.

Acknowledgements

The support of the UK Engineering and Physical Sciences Research Council (EPSRC) under the MORPHINE project (grant EP/N007379/1) is gratefully appreciated.

References

Arriaga, J., Rutten, J., Ribas, F., Falqués, A., and Ruessink, G., 2017. Modeling the long-term diffusion and feeding capability of a mega-nourishment. *Coastal Engineering*, 121: 1-13.

Atkinson, K.E., 1988. *An Introduction to Numerical Analysis*, John Wiley & Sons.

Brown, J.M., Phelps, J.J.C., Barkwith, A., Hurst, M.D., Ellis, M.A., and Plater, A.J., 2016. The effectiveness of beach mega-nourishment, assessed over three management epochs. *Journal of Environmental Management*, 184(2): 400-408.

Burcharth, H.F., Zanuttigh, B., Andersen, T.L., Lara, J.L., Steendam, G.J., Ruol, P., Sergent, P., Ostrowski, R., Silva, R., Martinelli, L., Nørgaard, J.Q.H., Mendoza, E., Simmonds, D., Ohle, N., Kappenberg, J., Pan, S., Nguyen, D.K., Toorman, E.A., Prinos, P., Hoggart, S., Chen, Z., Piotrowska, D., Pruszak, Z., Schönhofer, J., Skaja, M., Szmytkiewicz, P., Szmytkiewicz, M., Leont'yev, I., Angelelli, E., Formentin, S.M., Smaoui, H., Bi, Q., Sothmann, J., Schuster, D., Li, M., Ge, J., Lendzion, J., Koftis, T., Kuznetsov, S., Puente, A., Echavarri, B., Medina, R., Díaz-Simal, P., Rodriguez, I.L., Maza, M., and Higuera, P., 2015. *Chapter 3 - Innovative Engineering Solutions and Best Practices to Mitigate Coastal Risk*, Coastal Risk Management in a Changing Climate, Butterworth-Heinemann.

Capobianco, M., Hanson, H., Larson, M., Steetzel, H., Stive, M.J.F., Chatelus, Y., Aarninkhof, S., and Karambas, T., 2002. Nourishment design and evaluation: applicability of model concepts. *Coastal Engineering*, 47: 113-135.

Castelle, B., Turner, I.L., Bertin, X., and Tomlinson, R., 2009. Beach nourishments at Coolangatta Bay over the period 1987–2005: Impacts and lessons. Coastal Engineering, 56: 940-950.

Cooke, B.C., Jones, A.R., Goodwin, I.D., and Bishop, M.J., 2012. Nourishment practices on Australian sandy beaches: A review. *Journal of Environmental Management*, 113: 319-327.

de Schipper, M.A., de Vries, S., Ruessink, G., de Zeeuw, R.C., Rutten, J., van Gelder-Maas, C., and Stive, M.J.F., 2016. Initial spreading of a mega feeder nourishment: Observations of the Sand Engine pilot project. *Coastal Engineering*, 111: 23-38.

De Schipper, M.A., De Vries, S., Stive, M.J.F., De Zeeuw, R.C., Rutten, J., Ruessink, B.G., Aarninkhof, S.G.J., and Van Gelder-Maas, C., 2014. Morphological Development of a Mega-Nourishment: First Observations at the Sand Engine, *ICCE 2014*.

Dean, R.G., and Dalrymple, R.A., 2004. *Coastal processes: with engineering applications*, Cambridge University Press.

Farlow, S.J., 1982. *Partial Differential Equations for Scientists and Engineers*, John Wiley & Sons.

Hanson, H., 1987. *GENESIS-A generalized shoreline change numerical model for engineering use*. Dept. of Water Resources Eng. Lund, Lund Inst. of Tech/.Univ. of Lund.

Janssen, G., Kleef, H., Mulder, S., and Tydeman, P., 2008. Pilot assessment of depth related distribution of macrofauna in surf zone along Dutch coast and its implications for coastal management. *Marine Ecology*, 29: 186-194.

Kuang, C., Pan, Y., Zhang, Y., Liu, S., Yang, Y., Zhang, J., and Dong, P., 2011. Performance Evaluation of a Beach Nourishment Project at West Beach in Beidaihe, China. *Journal of Coastal Research*: 769-783.

Larson, M., Hanson, H., and Kraus, N.C., 1987. *Analytical solutions of the one-line model of shoreline change*, Technical Report CERC-87-15, USAE-WES, Coastal Engineering Research Cente.

Luijendijk, A.P., Ranasinghe, R., de Schipper, M.A., Huisman, B.A., Swinkels, C.M., Walstra, D.J.R., and Stive, M.J.F., 2017. The initial morphological response of the Sand Engine: A process-based modelling study. *Coastal Engineering*, 119: 1-14.

Luo, S., Cai, F., Liu, H., Lei, G., Qi, H., and Su, X., 2015. Adaptive measures adopted for risk reduction of coastal erosion in the People's Republic of China. *Ocean & Coastal Management*, 103: 134-145.

Nicholls, R.J., Wong, P.P., Burkett, V.R., Codignotto, J., Hay, J.E., McLean, R.F., Ragoonaden, S., and Woodroffe, C.D., 2007. *Coastal systems and low-lying areas. Climate change 2007: Impacts, adaptation and vulnerability*, Cambridge University Press.

Ojeda, E., Ruessink, B.G., and Guillen, J., 2008. Morphodynamic response of a two-barred beach to a shoreface nourishment. *Coastal Engineering*, 55: 1185-1196.

Pelnard-Considere, 1956. Essai de theorie de 1'evolution des forms de rivages en plaged e sablee t de galets. *4th Journees de l' Hydraulique, les Energiesd e la Mer*, Question III Rapport: 289-298.

Peterson, C.H., and Bishop, M.J., 2005. Assessing the environmental impacts of beach nourishment. *BioScience*, 55: 887-896

Reeve, D.E., 2006. Explicit Expression for Beach Response to Non-Stationary Forcing near a Groyne. *Journal of Waterway, Port, Coastal, and Ocean Engineering*, 132: 125-132.

Roberts, T.M., and Wang, P., 2012. Four-year performance and associated controlling factors of several beach nourishment projects along three adjacent barrier islands, west-central Florida, USA. *Coastal Engineering*, 70: 21-39.

Speybroeck, J., Bonte, D., Courtens, W., Gheskiere, T., Grootaert, P., Maelfait, J.-P., Mathys, M., Provoost, S., Sabbe, K., Stienen, E.W.M., Lancker, V.V., Vincx, M., and Degraer, S., 2006. Beach nourishment: an ecologically sound coastal defence alternative? A review. *Aquatic Conservation: Marine and Freshwater Ecosystems*, 16: 419-435.

Stive, M.J.F., Schipper, M.A.d., Luijendijk, A.P., Aarninkhof, S.G.J., Gelder-Maas, C.v., Vries, J.S.M.v.T.d., Vries, S.d., Henriquez, M., Marx, S., and Ranasinghe, R., 2013. A New Alternative to Saving Our Beaches from Sea-Level Rise: The Sand Engine. *Journal of Coastal Research*: 1001-1008.

Valsamidis, A., Cai, Y., and Reeve, D.E., 2013. Modelling beach-structure interaction using a Heaviside technique: application and validation. *Journal of Coastal Research*: 410 - 415.

Valsamidis, A. and Reeve, D.E., 2016. Modelling shoreline evolution in the vicinity of a groyne and a river. *Journal of Coastal Research*: 410 - 415.

van Duin, M.J.P., Wiersma, N.R., Walstra, D.J.R., van Rijn, L.C., and Stive, M.J.F., 2004. Nourishing the shoreface: observations and hindcasting of the Egmond case, The Netherlands. *Coastal Engineering*, 51: 813-837.

Villatoro, M., Silva, R., Méndez, F.J., Zanuttigh, B., Pan, S., Trifonova, E., Losada, I.J., Izaguirre, C., Simmonds, D., Reeve, D.E., Mendoza, E., Martinelli, L., Formentin, S.M., Galiatsatou, P., and Eftimova, P., 2014. An approach to assess flooding and erosion risk for open beaches in a changing climate. *Coastal Engineering*, 87: 50-76.

Walton, T. L. and T. Y. Chiu, 1979. A Review of Analytical Techniques to Solve the Sand Transport Equation and Some Simplified Solutions. *Coastal Structures '79*, ASCE.

Walton Jr, T.L., and Dean, R.G., 2011. Shoreline change at an infinite jetty for wave time series. *Continental Shelf Research*, 31: 1474-1480.

Yates, M.L., Guza, R.T., O'Reilly, W.C., and Seymour, R.J., 2009. Seasonal persistence of a small southern California beach fill. *Coastal Engineering*, 56: 559-564.

Zacharioudaki, A., and Reeve, D.E., 2008. Semianalytical Solutions of Shoreline Response to Time-Varying Wave Conditions. *Journal of Waterway, Port, Coastal, and Ocean Engineering*, 134: 265-274.