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THE USE OF DATA-DRIVEN TECHNIQUES FOR ANALYZING MESO-SCALE BEACH EVOLUTION ON THE SUFFOLK COAST (UK)

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Abstract

Predicting the meso-scale morphological evolution of our shorelines remains a major challenge for coastal management. The main aim of this paper is to use long-term coastal monitoring data of beach profile elevations within the Suffolk coast (east coast of the United Kingdom) to investigate meso-scale linkages between physical processes and coastal response. Patterns of beach behaviour are investigated using Empirical Orthogonal Function (EOF) and links between wave climate and beach response are explored through Canonical Correlation Analysis (CCA). The datasets consist of detailed nearshore wave measurements and profile surveys of the coastal frontage covering a period of over 22 years. The structure of the datasets and the data handling methods are described. Some results and interpretation of the analyses are presented. In particular we find a strong correlation between the first temporal eigenfunction of beach profiles and the trend in beach volume at three sites with very different morphological behaviours.

Key words: beach profile, canonical correlation analysis, data-driven technique, empirical orthogonal functions, statistical models.

1. Introduction

This paper concerns the use of long-term coastal monitoring data to investigate linkages between physical processes and coastal response over periods of several years. Information from coastal monitoring programmes is growing at exceptional rates due to government responses in the UK and elsewhere to climate change. The development of strategic approaches to coastal planning means there is global interest in advancing our understanding of evolution of coastal morphology over medium and long-term scales, (10-100 years). The information from coastal monitoring programmes can provide new scientific understanding, better prediction tools for coastal management and also improve process-based models. The acquisition of good quality, long term observations is fundamental and the continuation of monitoring programmes over periods of decades is critical as this increases the likelihood of a wide range of real-world situations being encapsulated in the observational database from which projections are made.

Sophisticated statistical methods for analysing patterns in one or more time series have been developed over recent years, taking advantage of the growing power of computers. These methods have been called 'data-driven' methods. Data-driven techniques include Empirical Orthogonal Function (EOF) and Canonical Correlation Analysis (CCA). EOFs provide a means of analysing a time history of observations of a quantity along a line or over a 2D grid so that the variability within the observations is split into those in space and those in time (Winant et al., 1975; Vincent et al., 1976; Wijnberg and Terwindt, 1995; Horrillo-Caraballo et al., 2015; Karunarathna et al., 2016). Larson et al. (1997) applied the method to beach profiles using the alongshore and cross-shore variability of the seabed and Reeve et al (2001) to analyse decadal morphological changes of offshore sandbanks on the east coast of the UK.

CCA is a bivariate analysis method which can describe patterns of behaviour between two sets of simultaneous measurements. In our application we are interested in the physical forcing due to waves and the beach response as measured by changes in profile. CCA provides a means of evaluating the importance of interaction among these observations (Larson et al., 2000; Horrillo-Caraballo and Reeve, 2008). With

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coastal monitoring programmes having been set up as part of coastal management planning in many parts of the world there are now a growing number of sites where datasets of several decades have been compiled. These provide a huge resource, not only for coastal managers but also for academic research into meso-scale beach response. They are also of sufficient length for sophisticated statistical tools to be employed to probe the datasets for patterns of behaviour and linkages between forcing and response at the meso-scale, (10's of kilometres, months to years).

The data used for this study covers part of the east coast of the UK; the Suffolk coastal frontage from Lowestoft south pier to Felixstowe Landguard Point (Figure 1), which is approximately 72km in length. This coast is characterised by soft eroding cliffs, shingle beaches and coastal lagoons and includes some estuaries. Rising sea levels and the soft nature of the coastline makes it susceptible to erosion and the adjacent hinterland increasingly vulnerable. There is a strong longshore component to sediment movements on this coastline, (typical longshore transport rates being in the range of 20000 to 60000 m^3/year), and so we anticipate a reasonable correlation between wave activity and beach movements. The majority of beaches in the area have been able to behave naturally and erode/accrete in response to natural processes as a result (EA, 2011).

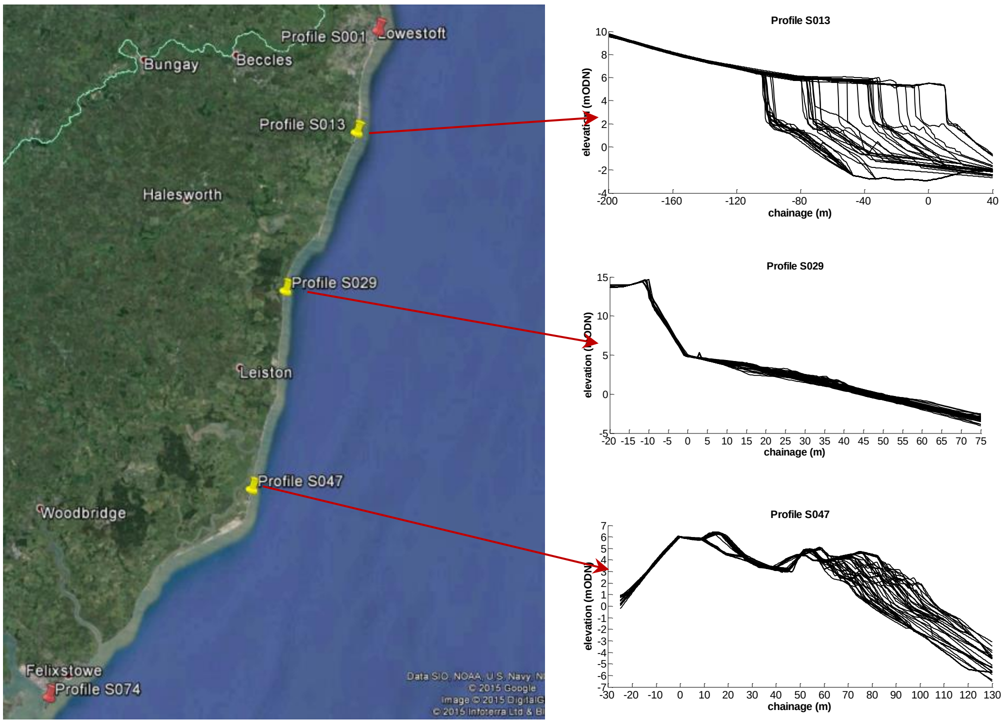

Figure 1. Location of the Suffolk study site (red markers are the limit of the area, yellow markers are the profile locations) and plan view of profile evolutions in time (time from bottom to top).

2. Field data

2.1 Beach profiles

As part of the strategy established by the UK government through the shoreline management plans and the long-term monitoring beach survey programme, the UK Environment Agency have been measuring beach profiles in the Suffolk area since 1991, twice a year. The period covered by the measurements runs from August 1991 to September 2012. Bed levels have been recorded along 74 profiles lines spaced at approximately 1km intervals alongshore (Figure 1). Usually each profile extends across the intertidal zone and covers the backshore, foreshore and at times the nearshore. Profile surveys included in the analysis are the ones that go from the backshore to the water depth of the MLWL (Mean Low Water Level). The analysis of the data is not affected by the surveys, as they do not reach the depth of closure but reaching conclusions that concern cross-shore sediment transport are less certain. Following the procedure described by Li et al. (2005), the profile surveys were interpolated to a regular spacing of 0.5m resolution. The Ordnance Datum Newlyn (ODN) is used as a reference system for all the measurements.

Here we focus on three beach transects that illustrate the range of characteristics exhibited by the shoreline along this length of coastline. Profile S013, (Figure 1 - top right panel), has an intertidal beach backed by an eroding cliff. Profile S029, (Figure 1 - middle right panel) is backed by a cliff that has historically eroded, but is currently stable. Profile S047, (Figure 1 - bottom right panel) is located on Sudbourne beach, extending north from Orford Ness, a barrier system comprising distinct beach ridges.

The maximum, mean and minimum of the subset of measured profiles along transects S013, S029 and S047, respectively, which were used in the analysis are shown in Figure 2.

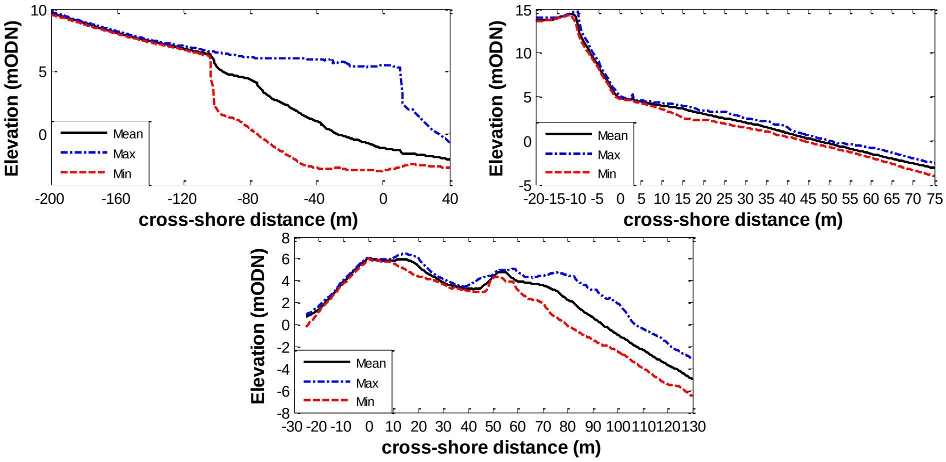

Figure 2. Maximum, mean and minimum for profiles S013 (top left panel), S029 (top right panel) and S047 (bottom panel) which are used for this study. Note variable axis scales.

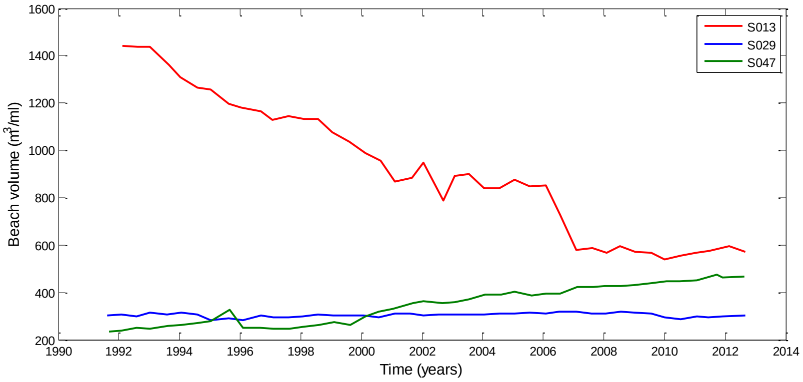

Figure 3. Beach volumes for profiles S013, S029 and S047.

The profiles at these three locations show very different behaviour as may be seen from the beach volumes at each transect, (Figure 3). Profile S013 shows an eroding tendency and the cliff face has retreated around 120m. Changes at profile S029 are considerably smaller than that of the previous profile; mostly these changes are related to morphological shifts in the foreshore. The position of the cliff face is maintained and only changes in the beach area are evident. The middle and bottom sections of the beach are the more noticeable changes detected in bed levels. A different behaviour with respect to the previous ones is shown by profile S047. The backshore beach ridge environment is mostly constant. This profile presents a consistent accretional tendency that was interrupted during the years 1994 to 1996, in which there was accretion during 1994 and erosion during the winter period of 1995-1996 (Figure 3).

2.2 Wave data

Wave data has been obtained from the wave hindcast model from the UK Meteorological Office (CEFAS, 2013) for different points around the Suffolk coast. The wave data obtained from the model has been compared with wave buoy data around UK. Bradbury et al. (2004) found a strong correlation between measured and modelled conditions. They concluded that confidence in the model data and the wave buoy measurement was extremely high. There is no so much difference between the three wave model points considered on the study. The wave direction is very bimodal in direction; waves are predominately from the NNE or the south.

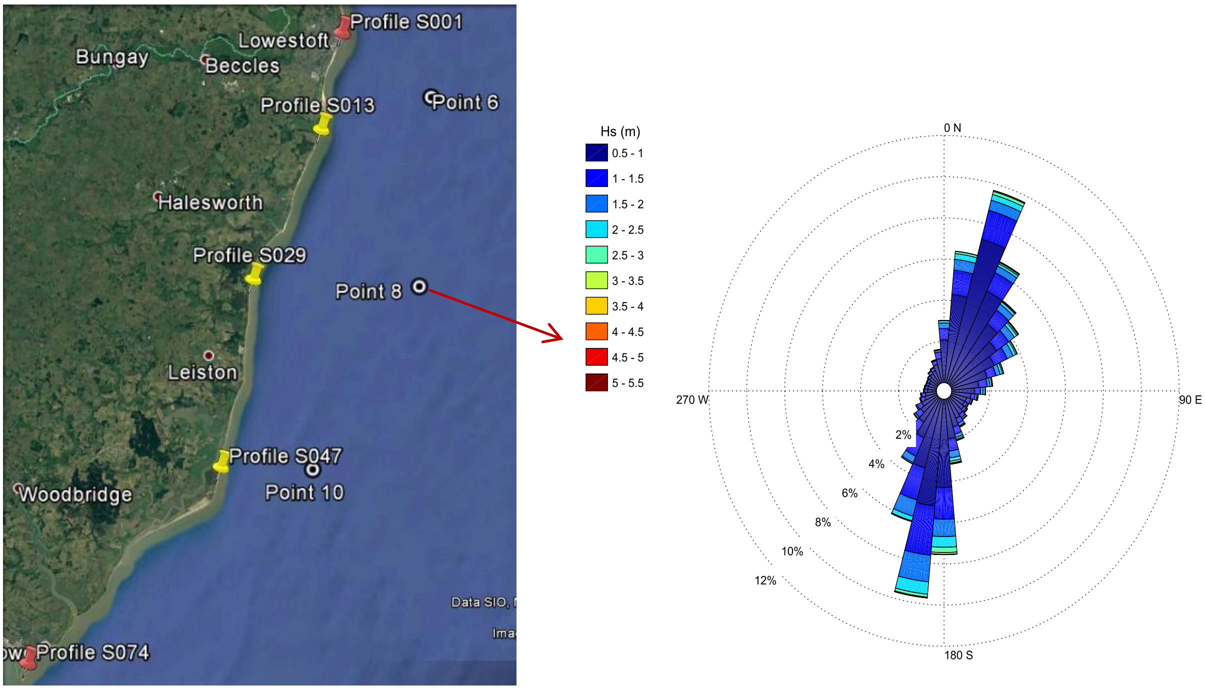

Figure 4. Wave rose for a location near the middle of the study site.

3. Methodology

3.1. *Empirical Orthogonal function (EOF)*

The EOF technique has been widely used in coastal studies and the method is generally well-known and the interested reader may find the details in the references cited earlier. Briefly, the EOF method separates the spatial and temporal variability in a set of measurements and is quite similar to the Fourier analysis; with two important differences: the EOF can be used in data that is not regularly spaced in time; the form of the function is not defined, but is designated by the nature of the data itself.

The discrete beach level measurements can be denoted by $g(\xi_l,t_k)$, where $1 \leq l \leq L$ and $1 \leq k \leq K$. The idea of EOF analysis is to express the data as,

$$g(\xi_l,t_k)=\sum_{p=1}^{L} c_p(t_k).e_p(\xi_l) \tag{1}$$

where:
e_p : eigenfunctions of the square L x L correlation matrix of the data
c_p : coefficients describing the temporal variation of the p^{th} eigenfunction.

The correlation matrix, A, is calculated directly from the data and its elements are in the form of,

$$a_{mn} = \frac{1}{L.K} . \sum_{k=1}^{K} g(\xi_m, t_k).g(\xi_n, t_k) \tag{2}$$

A is real and symmetric and has L real eigenvalues, λ_p, with $1 \leq p \leq L$. The L corresponding eigenfunctions, $e_p(\xi_l)$, satisfy the matrix equation

$$Ae_p = \lambda_p e_p \tag{3}$$

The eigenfunctions of a real L x L symmetric matrix are mutually orthogonal. It is standard to normalise the eigenvectors so they have unit length;

$$\sum_{l=1}^{L} e_p(\xi_l).e_q(\xi_l) = \delta_{pq} \tag{4}$$

where δ_{pq} is the Kronecker delta. From Equations (1) and (4), the coefficients c_p may be calculated from

$$c_p(t_k) = \sum_{l=1}^{L} g(\xi_l, t_k).e_p(\xi_l) \tag{5}$$

3.2. *Canonical Correlation Analysis (CCA)*

A detailed description of the CCA method may be found in Clark (1975) and Różyński (2003). The CCA technique is used to study bivariate datasets to determine intercorrelations. As a noise-reduction method prior to CCA it is common to apply the EOF technique to each of the two datasets in turn to create filtered data sets. (Finding the EOF expansion as per (1), then truncating the series and reforming the dataset).

If the two original data sets are denoted Y, (wave height - matrix size: $nt \cdot ny$), and Z, (time sequence of beach profiles - matrix size: $nt \cdot nz$), a regression matrix can be derived between the two matrices representing the correlation between the dominant patterns in the two variables established. This means that if the variable Y is known for some future time, the other variable Z can be predicted by using the regression matrix (Larson et al., 2000). The predictions, Z_p, given a predicted wave matrix Y_p is given by:

$$Z_p = Y_p A \tag{6}$$

where A is a regression matrix that defines the relationship between the two variables based on historical measurements. Equation (6) has been proposed as the basis of a data-driven predictive algorithm for shoreline morphology based on estimates of future wave conditions by Horrillo-Caraballo and Reeve (2010) and Horrillo-Caraballo et al. (2016).

4. Data Analysis

4.1. *Pre-processing and Analysis*

The dates of beach surveys at the three different locations transects are given in Table 1. There are around 43 measurements for each of the profiles. The wave data consisted of approximately 130000 consecutive values of integrated wave parameters (H_s, T_z, direction). The set of profiles is used to determine the EOF and the set of profiles and waves are needed to perform the CCA.

Table 1. Summary of the data used for the EOF and CCA analysis

Profile	Period considered (dd/mm/yyyy)	No. of records
S013	13/02/1992-07/09/2012	42
S029	16/08/1991-06/09/2012	43
S047	08/09/1991-24/08/2012	43

Two time series with an equal number of measurements are needed for the application of the CCA technique. In order to have two series of equal length, wave measurements have been collected to generate a sequence of probability density functions (pdfs) of wave height in our case. This methodology was proposed by Larson and Kraus (1995) using a parametric form of pdf. More recent studies done by Rihouey (2004) and Horrillo-Caraballo and Reeve (2008) have suggested that by using an empirical distribution for the pdfs a better performance is obtained. The wave records were used to compile empirical pdfs for the CCA analysis and were divided into about 200 intervals in wave height to provide an adequate resolution over the range of wave heights.

Figure 5 shows the wave height raw data for Point 8 and the corresponding composite pdf for the significant wave height. The density function shows peaks at small wave heights, (~0.6m), the curve falls rapidly for larger wave heights, with the maximum wave height of 5.2 recorded during the period occurring once. The density functions assembled for each period between two beach profile measurements are used to create the matrix *Y* for the CCA. The beach profiles are used to construct the matrix Z.

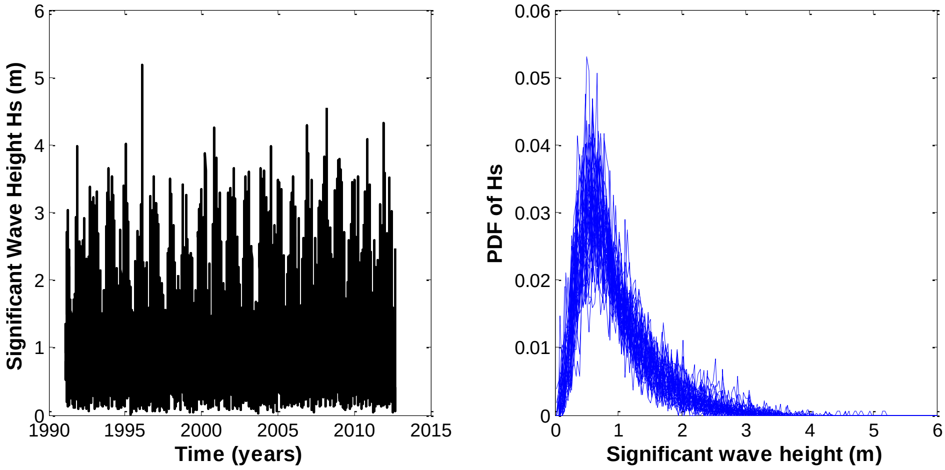

Figure 5. Time series of significant wave height – Hs (left panel) and time series of empirical probability density function (right panel) for wave point 8.

4.2. *Analysis of the Data*

CCA was applied to profiles S013, S029 and S047 using the wave pdfs. The analysis consists of using the wave pdfs and beach profile measurements to define the regression matrix for each of the beach profile. For each case, EOF expansion and truncation were used in order to pre-filter the data sets before performing the CCA analysis.

5. Results

The analyses were performed as described above. EOF analyses were performed as part of the filtering process for the CCA. Results from the EOF analysis are shown first, followed by those of the CCA.

Using the “rule of thumb” (North et al., 1982), three to five EOF modes are appropriate to represent most of the variation in the data sets. In general, as the mode number increases, so the number of peaks and troughs in the modal function increases, and its contribution to the total variance in the datasets decreases. At some stage, the shapes of the modes will cease representing well-defined patterns and may be considered effectively as ‘noise’.

The first three spatial EOFs (Figure 6a) obtained from the profile S013 explained 86% of the variation in the data (E1: 55%, E2: 25% and E3: 6%), for profile S029 (Figure 6b) 68% of the variation is explained (E1: 30%, E2: 23% and E3: 15%) and profile S047 (Figure 6c) explained 69% of the variation in the data (E1: 36%, E2: 22% and E3: 11%). In all cases, the time mean was subtracted before analysis in all data sets. For the three sites, the EOFs describing the profile shapes are quite complex due to the different configuration for each of the profiles, and this explains the relatively low values of explained variance in the first three EOFs. Comparing the EOFs of the profiles (Figures 6a, 6b and 6c), there are clear differences. The EOFs for profile S013 (Figure 6a) show a stable behaviour between the chainage -200m to -100m and strong variability between chainages of -100 m to the end of the profile indicating a zone very active. According to the survey data this profile has retreated approximately 120m in around 20 years (6m/year). For profile S029 (Figure 6b) the stable behaviour of the EOFs is seen in the first 10m of the profile, over the top of the cliff area of the profile; this area does not experience too many changes in comparison with the swash area. The first EOF at S029 shows an increase in its amplitude from negative values to positive values, this is due to the variation in this area with respect to the mean value. The second EOF can be related to the changes in the bar position over the profile and the third EOF may be related with exchange of material across the profile during storm conditions. In contrast profile S047, shows a decrease in the magnitude of the first EOF in comparison with the others two that show an increase in the magnitude of the first EOF. That gives an explanation that the first two profiles tends to erode and the third one (S047) tends to accrete.

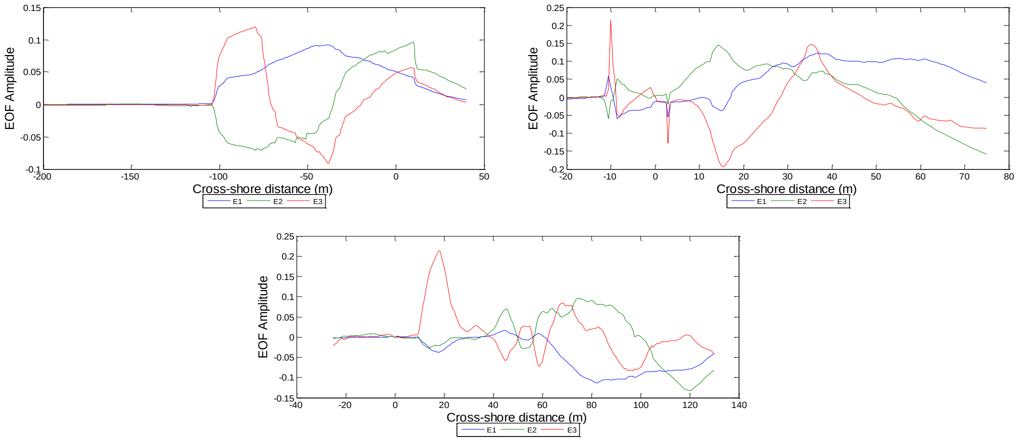

Figure 6. First three EOFs (E1 to E3) determined from measured (a) beach profiles S013, (b) S029 and (c) S047.

EOF analysis was also performed on the wave height pdf’s but do not have such a direct interpretation as the beach profiles. The EOFs associated with the wave height pdfs, (Figure 7), reflect variations across the distribution of wave heights (see Figure 5b). Larson et al. (2000) interpret these variations as coming from changes of season, episodic storms and changes in swell characteristics linked to distant storms coming from far away.

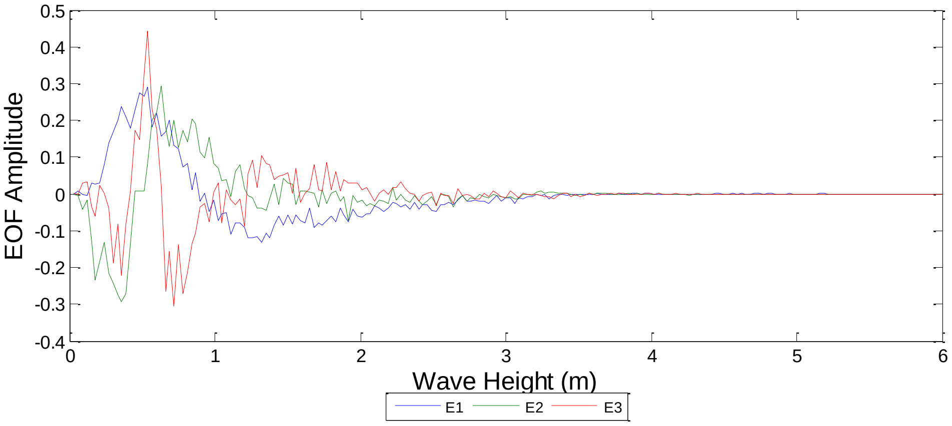

Figure 7. First three EOFs (F1 to F3) determined from wave Point 8.

The temporal eigenfunctions describes the temporal variation in the profiles and in the wave heights. Every temporal eigenfunction corresponds to a spatial eigenfunction. All temporal eigenfunctions are normalised, defining how the amplitude of the corresponding spatial eigenfunction varies throughout the period covered by the surveys (Figures 8 and 9).

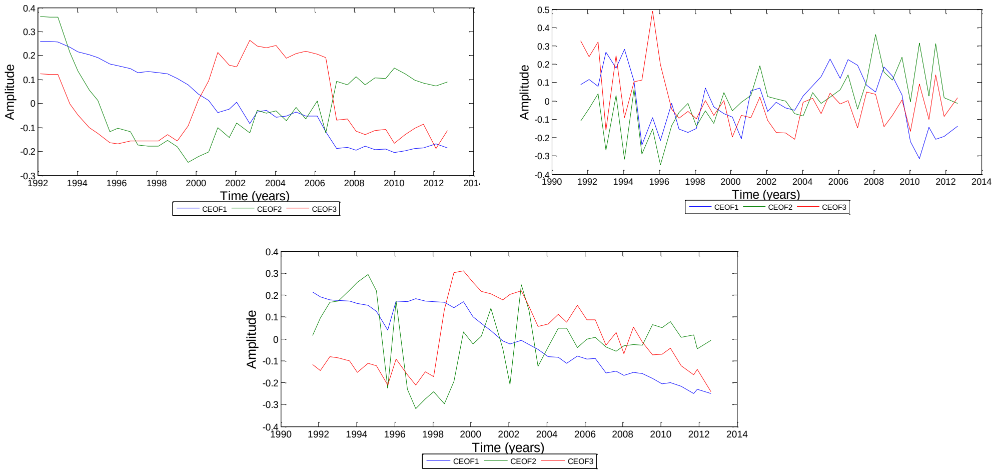

Figure 8. First three temporal EOFs of beach profiles (a) S013, (b) S029 and (c) S047.

Figure 8 shows the temporal eigenfunctions for the first three spatial eigenfunctions for each of the three profiles in the study. As it was mentioned earlier, the mean has been removed from the data and the first temporal mode does not therefore correspond to the mean. A striking feature of the results is that the first temporal eigenfunction exhibits a similar behavioural trend as the profile volume for each of the profiles. The trend for profile S013 goes from positive values in 1992 to negative values at the beginning of the 2000s and from 2007 the behaviour of the trend is quite stable. An interpretation for this behaviour can be the continuing erosion in this part of the study site. The remaining eigenfunctions exhibit variations between positive and negative values between the periods of the study. The first temporal EOF mode for profile S029 goes from negatives values to positives values and return again from positives to negatives with the same behaviour as the profile volume. This temporal EOF mode illustrates the change of volume occurring on the profile. For profile S047, the first temporal EOF mode also follows the same trend as the profile volume but in a mirroring way (Figure 3). That is to say the erosion at Profile S013 manifests itself as the spatial eigenfunction being positive while the temporal eigenfunction goes from positive to negative over the period. The net contribution of this to the summation in equation (1) is to build the profile in the early part of the measurement period and to erode it in the later part – corresponding to the overall erosional trend. However, at Profile S047 the spatial eigenfunction is predominantly negative and the temporal eigenfunction goes from positive to negative over the period. The consequent contribution of this to the summation in equation (1) is to erode in the early part of the period and build the profile in the later part – corresponding to the accretive trend observed in the sequence of profiles.

The temporal eigenfunctions for the first three spatial eigenfunctions for the wave data Point 8 is shown in Figure 9. The temporal EOFs show little obvious structure apart from a ‘saw-tooth’ oscillation. This could be interpreted as a seasonal cycle but this is also close to the Nyquist frequency of the sampling rate so more frequent observations would be required to verify this hypothesis.

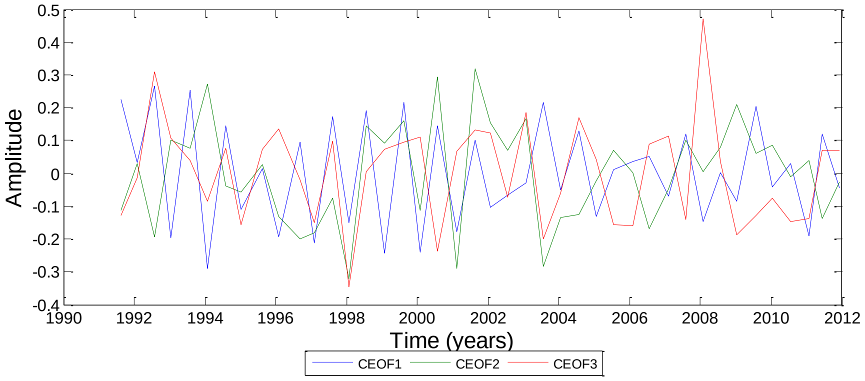

Figure 9. First three temporal EOFs (CEOF1 to CEOF3) for the wave height pdf at Point 8.

The CCA analysis was performed for each of the three profiles and the wave height distribution. Applying CCA for the profiles and wave height pdfs produced a maximum correlation of 0.95 between U1 and V1 - temporal amplitudes of the first CCA modes (See Figure 10a). Figures 10b and 10c display the CCA modes and indicate that movement of material between the cliff face and the low water level is associated with an increase in the probability of higher waves in the pdf and vice versa. The erosion will occur in the inshore section due to higher waves and the material will be deposited in the area of the bar, if this bar exists.

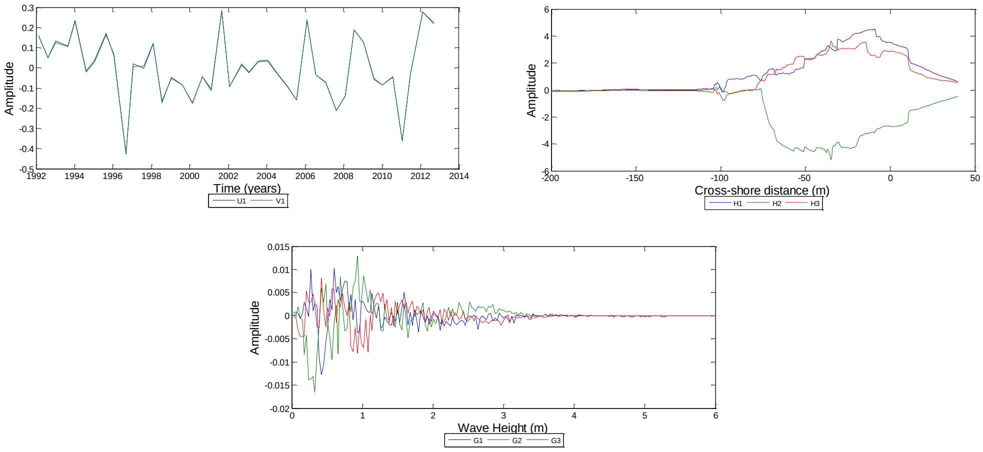

Figure 10. Results of the CCA analysis between beach profile S013 and wave height pdfs from Point 8: (a) temporal amplitudes of the first CCA modes (U1 and V1), (b) spatial amplitudes of the first three CCA modes for profile elevation (H1 to H3), and (c) wave height amplitudes of the first three CCA modes for wave height pdf (G1 to G3).

Mode H1 shows the profile elevation that is related with variations in the composite wave pdf as given by G1. Accordingly, H1 implies a general decrease across the profile when G1 causes a decrease or increase in the wave pdf. The higher modes H2 and H3 are associated with more complex changes in the wave pdf determined by G2 and G3.

6. Conclusions

The importance of monitoring and collecting good quality data has been widely recognised for informing coastal management. As the ambitions of management extend to developing strategic planning up to 100 years ahead, so the importance of long term data sets increases. This is not only because longer term cycles and trends require records over longer periods to establish their presence or otherwise, but also to provide appropriate datasets against which to calibrate and validate predictive morphological models. As noted by Reeve et al (2016) the importance of continuing existing monitoring programmes is paramount if our understanding of meso-scale processes is to be improved and reliable meso-scale models are to be developed.

The data-driven methods discussed in this paper, and applied to real data, are only a small part of the armoury of statistical methods that can be brought to bear on this problem. In this paper we have applied two data-driven techniques (EOF and CCA) at three different sites in the Suffolk coast in order to investigate patterns in the spatial and temporal variability of its morphology at which there are coincident records of beach profiles and wave conditions for over 20 years. The three different profiles used in this study present quite dissimilar characteristics: the one to the north is eroding, the one in the middle area is more or less stable and the last one to the south is continuously accreting. The methods effectively analysed eroding, accreting and stable beach types and we found that the first temporal EOF correlated very well with the beach volume trends at each site, suggesting that this could be a useful proxy for representing beach health.

It has been established (Larson et al. 2000; Horrillo-Caraballo and Reeve 2008) that wave height distribution and beach profiles are a good metric to investigate processes and how the beach responds to these processes. The CCA technique, used in conjunction with EOF technique to reduce noise in the data, seems well suited for identifying patterns of variations in the wave and profile data. However, it is important to point out that the correlation between the temporal amplitudes of the CCA modes of wave height distributions and beach profiles is high enough to indicate the existence of a well-defined cause-and-effect relationship. Though, this relationship cannot be completely solved by a static model as the one utilised in this study, it could very well be described by a low-dimensional state-space model i.e., a model that would not include any or little of the process dynamics. Viewed from this perspective analyses such as those in this paper offer a good way to generate hypotheses for developing meso-scale modelling of shorelines.

Acknowledgements

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