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EQUILIBRIUM PLANFORM OF HEADLAND BAY BEACHES: EFFECT OF DIRECTIONAL WAVE CLIMATE

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Abstract

Equilibrium beach formulations are useful tools for providing solutions to coastal erosion problems, defining the final orientation of a beach on a scale of years and thus they are used for evaluating the shoreline response due to human interventions. Throughout the literature, several equations can be found for obtaining the Static Equilibrium Planform (SEP) of Headland Bay Beaches (HBBs), one such being the Parabolic Bay Shape Equation (PBSE). The SEP significantly depends on the location of the down-coast control point (P_o) which is the down-drift limit from which the PBSE is applicable. Existing formulae for determining the location of (P_o), are limited to specific conditions, particularly for close structures from the shoreline. Consequently, they are neither valid for (HBBs) in zones with a wide variability of wave climate directionality nor in cases where the diffraction point is located far from the shoreline. Accordingly, this study investigates the effect of the directional wave climate on locating the (P_o) point, employing 44 (HBBs) in Spain and Latin America. The results of the study have confirmed the dependence of the location of (P_o), and thus the planfrom shape on the degree of variability of the nearshore directional wave climate.

Key words: Headland bay beaches, wave diffraction, wave climate, equilibrium planform, down-coast control point, directional variability

1. Introduction

Many coastline sections feature curved shoreline geometry behind natural headlands and man-made breakwaters. Headland Bay Beaches (HBBs) exemplify one of the most common physiographic features on the oceanic margins all over the world. It is claimed that they occupy about 50% of the world's coastlines (Inman and Nordstrom, 1971). They also exist on the edge of coastal closed seas and lakes (Hsu et al., 2010). This coastal feature is considered by both coastal scientists and engineers to be a stable landform. Because of their geometries, these shorelines are also referred to as embayed beaches or structurally controlled beaches (Short and Masselink, 1999), pocket beaches (Yasso, 1965), spiral beaches (Krumbein, 1944) and crenulate-shaped bays (Silvester and Ho, 1972). The stability of these beaches, with their famous curved parts, have inspired research by coastal engineers to study and define them in the planform in relation to the wave climate. Hsu et al. (2010) classified the planform of (HBBs) to be in static equilibrium, dynamic equilibrium, unstable or natural reshaping. A static equilibrium HBB is a state where the predominant waves are breaking simultaneously around the whole bay periphery; hence the littoral drift produced by longshore currents is almost non-existent and no additional sediment is required to maintain the long-term stability.

Several empirical equations have been formulated to mimic the Static Equilibrium Planform (SEP) of natural headlands sculptured by nature. The most well-known models in the literature are the logarithmic spiral model (Krumbein, 1944; Yasso, 1965), the hyperbolic tangent model (Moreno and Kraus, 1999), and the Parabolic Bay Shape Equation (PBSE) derived by Hsu and Evans (1989). Nowadays, the PBSE is the most widely used model in coastal engineering practices (González et al., 2010), and has received the recognition of the Coastal Engineering Manual (USACE, 2002) for coastal management and project evaluation. Consequently, it has been implemented in the MEPBAY software (Klein et al., 2003b) as well as the Coastal Modeling System package (SMC) (González et al., 2007; González et al., 2016). The PBSE

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is a second-order polynomial equation derived from fitting the planform of 27 mixed cases of prototype and model bays believed to be in static equilibrium as explained in Figure 1. One of the unknowns in the application of the PBSE is locating the down-drift limit from which it is applicable, determining the affected part of the beach dominated by refraction-diffraction due to the presence of the headland structure, (González and Medina, 2001; Lausman et al., 2010 a, b). Based on the best fit of the SEP of 26 beaches along the Mediterranean and Atlantic coasts of Spain, González and Medina (2001) provided a methodology for defining that limit, denoted as the (P_o) point, see Figure 1, proposing the concept of the (α_{min}) angle given as:

$$\alpha_{min} = \arctan\frac{X/L}{Y/L} = \arctan\frac{\sqrt{\frac{\beta_r^4}{16}+\frac{\beta_r^2}{2}\frac{Y}{L}}}{Y/L} \tag{1}$$

Where (β_r) is the distance parameter with a value of ($\beta_r = 2.13$) based on the best fit of the (SEP) of the 26 beaches and (Y/L) is the dimensionless distance between the diffraction point and the straight segment of the shoreline. This angle defines the location of the down-coast control point (P_o) which is the point that differentiates between the part of the beach affected by the headland structure where wave height gradients start in the transition and shadow zones due to the diffraction process, and the non-affected part of the beach where no longitudinal wave height gradients exist due to the coastal barrier. They applied the energy flux approach to locate the (P_o) point, stating that the straight segment of the SEP of a HBB is parallel to the wave front corresponding to the direction of the mean wave energy flux (θ_{EF}) at the diffraction point, which was also recommended by Hsu et al. (2010). It worth noting that the (α_{min}) angle was originally derived using the analytical solution of monochromatic wave diffraction for a flat bottom given by Penny and Price (1952), see Dean and Dalrymple (1991) and González (1995). According to González and Medina (2001), the (α_{min}) angle and thus the location of the (P_o) point is only dependent on the dimensionless distance (Y/L), referring to equation (1). The scaling wave length (L) is based on the wave period associated with the significant wave height exceeding 12 hours per year (H_{s12}) and the mean water depth along the wave crest at the diffraction point, see González and Medina (2001). However, that methodology was derived based on studies of beaches exposed to wave climates with almost a clear dominant direction, i.e. waves arrive at the beach from a narrow fan of directions and close diffraction points from the shoreline to be only valid within the domain (Y/L < 10).

Accordingly, the study hypothesizes that the (α_{min}) approach cannot be used for pocket beach cases with diffraction points far from the straight segment of the equilibrium shoreline and/or in cases with a wide variability of wave climate directionality close to the diffraction point. Consequently, the aim of this study is to check the work hypothesis and to investigate the influence of the nearshore wave climate directionality on the location of the (P_o) point and the (SEP) using real field cases and long wave data time series.

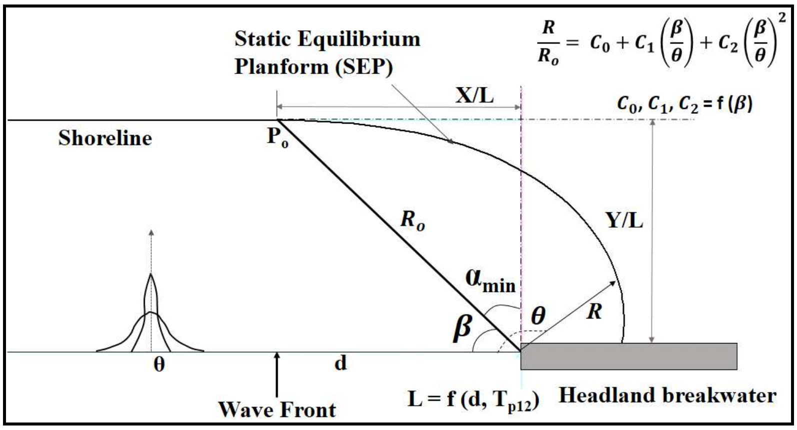

Figure 1. Definition sketch of the PBSE and the (SEP) for a (HBB) clarifying the (α_{min}) angle and the (P_o) point

2. Methodology and Tools

In order to achieve the target of the study, prototype cases of HBBs from Spain and Latin America were collected. This was followed by the analysis of the directional wave climate at the diffraction points of the selected cases. The planform shape obtained using the methodology of González and Medina (2001) as well as the best fit SEP of each embayed beach were then determined employing the SMC software. The results of the two plotted shapes of the planforms were compared and analyzed. Finally, the discussion of the results and the conclusions of the study are posted.

2.1. *Coastal Modeling System (SMC)*

The Coastal Modeling System (SMC) is a user-friendly software package developed by the University of Cantabria (UC) for the Dirección General de Costas (State Coastal Office) of the Spanish Environmental Ministry, see González et al. (2007). It includes some numerical models that allow the application in coastal projects of the methodologies and formulations proposed in several manuals elaborated for the ministry. The latest version of the system (SMC 3.0) is structured in a manner dividing the numerical models and data into two main tools, namely: (SMC-Tools) and (SMC Model). The former incorporates 3 modules: (a) IH-DATA, (b) IH-AMEVA and (c) IH-DYNAMICS, while the SMC model includes both short-term and long-term modules for studies on a scale of hours to days and a scale of years, respectively, in addition to a terrain module. It also includes a graphical user interface that permits stability testing and/or designing new beaches utilizing the “equilibrium beach” concept, which combines different equilibrium profile and planform formulas, see more details in González et al. (2007) and González et al. (2016).

The IH-DATA module (Gomes and Silva, 2014; González et al., 2016) has three databases. One is associated with time series of coastal waves called DOW (Downscaled Ocean Waves). The other two databases are associated with sea level time series: one is for astronomical tides called GOT (Global Ocean Tides), see González et al. (2016), and the other is for storm surges or meteorological tides called GOS (Global Ocean Surges), see Cid et al. (2014). These databases were generated over a long period of more than 60 years (from 1948 onwards), using re-analysis of wind fields and satellite data.

The IH-AMEVA module is used to statistically analyze the IH-DATA consisting of wave climates and sea level time series, and later for the statistical characterization of the results. Finally, the IH-DYNAMICS module is used in the post-processing stage, providing extensive data and results. It calculates the wave mean energy flux, littoral sediment transport, run-up and flooding levels in addition to climate change impacts on the coast.

In this study, the long-term module was used in the analysis of the directional wave climates in order to obtain the best fit equilibrium planform shape of real beach cases in the long term.

3. Study Cases and Data

A description of the employed beach cases, the available wave data and analysis as well as the applied procedure with the available tool are given in the subsequent sections.

3.1. *Beach cases*

This study analyzed the planform of 44 embayed beaches along the coasts of Spain, Brazil and Uruguay as shown in Figure 2. The selection of the beach was carefully carried out according to specific conditions; in particular, static equilibrium embayed beaches were chosen, both man-made and natural. These beaches have diffraction points with varying distances from the shoreline. They exemplify fully developed beaches (González and Medina, 2001) with a clear straight orientation in the planform. Moreover, the wave climates close to the diffraction points display different degrees of directional variability. In the current study, vertical aerial images of beaches were employed using the SMC-Tools module of the Coastal Modeling System (SMC) which captures vertical photos of beaches based on Google Earth imagery.

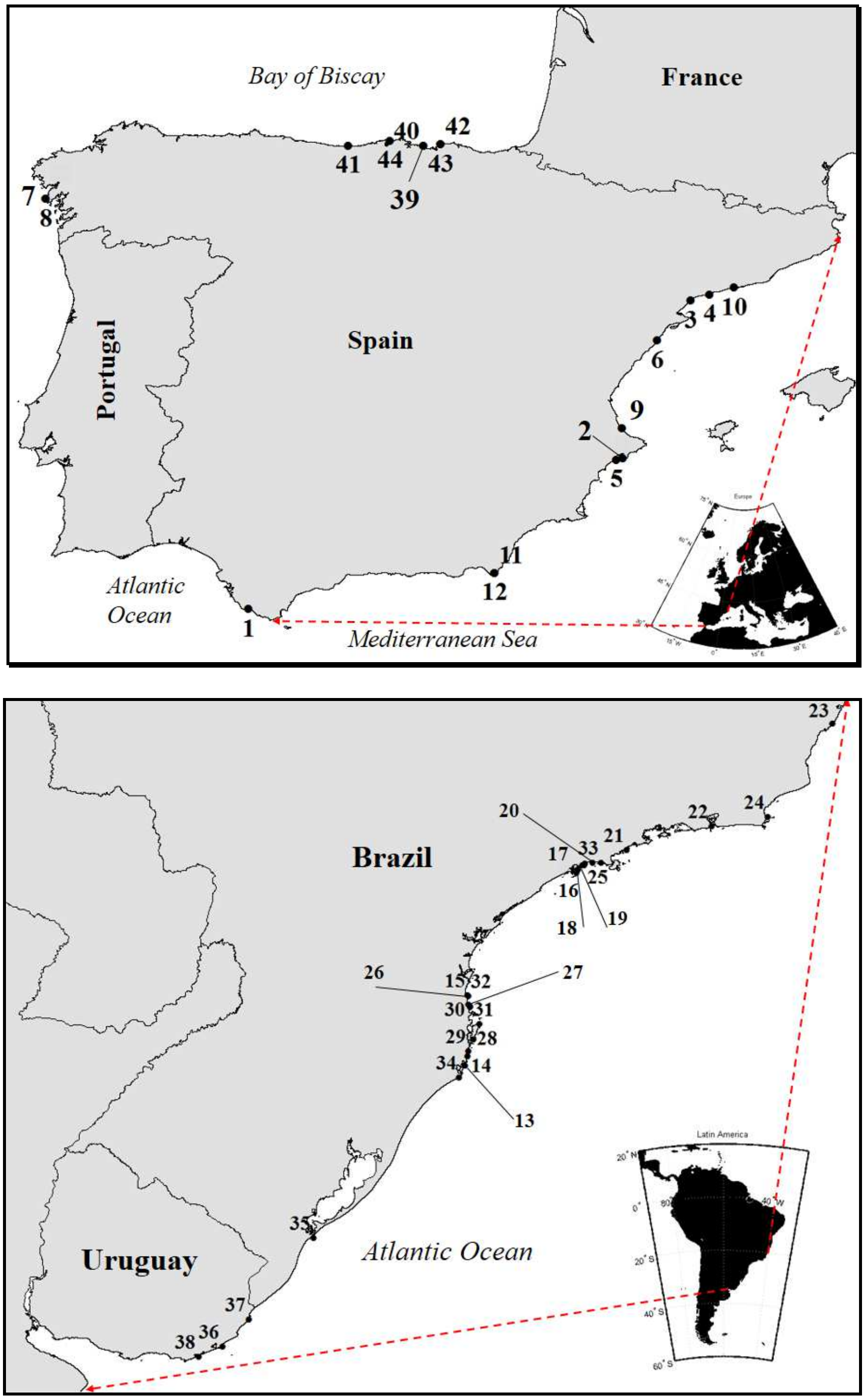

Figure 2. Location of selected headland bay beaches in static equilibrium along the coasts of Spain (upper panel) and Latin America (lower panel)

3.2. *Bathymetric and wave data*

Bathymetries of the coastal zones of Spain, Brazil and Uruguay collected by the Environmental Hydraulics Institute (IH Cantabria) were used in this study. These digitalized bathymetric data are incorporated in the IH-Data SMC-Tools module of the Coastal Modeling System (SMC) for littoral areas of the entire Spanish and Brazilian coasts as well as for the northern Uruguayan shores.

Regarding the data of the waves, the study adopted the hindcast wave time series of the DOW

(Downscaled Ocean Waves) database (Camus et al., 2013) representing a period more than 60 years (from 1948 onwards). The DOW database is a historical reconstruction of coastal waves. In other words, it is a downscaled wave re-analysis of coastal zones from a Global Ocean Waves (GOW) database (Reguero et al., 2012). The GOW was generated using the WAVEWATCH III model (Tolman, 1992) forced by the (NCEP/NCAR) wind field re-analysis, for more details see Reguero et al. (2012). The GOW database was then directionally calibrated using satellite data to avoid deviations and bias in the results, see more details described in Minguez et al. (2011a). This calibrated (GOW) data set was used to select a representative subset of sea states in the deepwater which guarantees that all possible conditions are represented, including extreme events, see Camus et al. (2011b). The selected sea states were propagated using the SWAN spectral wave model (Booij et al., 1999) with high spatial resolution over detailed bathymetries. Finally, the time series of the propagated sea state parameters at each location were reconstructed, see Camus et al. (2011a). The DOW wave climate database is available for the entire Spanish and Brazilian coasts, as well as the northern coasts of Uruguay with a high spatial resolution (0.01˚, i.e. each 1 km) along the coastlines. It provides different wave parameters for each sea state (e.g. the significant wave height H_s, spectral peak period T_p, mean wave direction θ_m, etc) with a temporal resolution of one hour.

3.3. *Analysis of wave climate*

For each embayed beach of the cases selected for this study, the wave climate close to the diffraction point of the protruding headland was analyzed, calculating the most important wave parameters required for this study. Using the DOW database, waves were characterized by calculating the energy flux (*EF*) for each sea state as the product of the wave energy (*E*) and the group celerity (C_g) as:

$$EF=E*C_g=\frac{1}{8}*\rho*g*H_s^2*C_g \tag{2}$$

Where ρ is the water density, g is the gravitational acceleration and H_s is the significant wave height. The significant wave height exceeding 12 hours each year (H_{s12}) and its corresponding spectral peak period (T_{p12}) were utilized as descriptors of the wave climate. This wave height is associated with a 99.86% exceedance percentile, representing a quantile of the high range wave heights and energy conditions during the year. Also, the wave length (*L*) close to the diffraction point for each bay beach was obtained as a function of both the wave period (T_{p12}) and the mean water depth (*d*) along the wave front close to the breakwater tip. This was used to scale the offshore distance between the diffraction point and the straight part of the shoreline, obtaining the (*Y/L*) ratio.

The direction of the mean wave energy flux (θ_{EF}) was considered as a directional descriptor of the wave climate. This variable is very important for the coastal system as it is significantly related to the change forms of beaches. It was calculated for the whole wave climate as:

$$\theta_{EF}=arctan\frac{F_y}{F_x}=arctan\frac{\sum_{i=1}^{n}F_i sin\theta_i}{\sum_{i=1}^{n}F_i\; cos\theta_i} \tag{3}$$

Where F_i and θ_i are the value and the direction of the wave energy flux for each sea state, respectively.

In order to consider the directional variance of the wave climate and its effect on the equilibrium planform of beaches, the standard deviation of the mean energy flux direction ($\sigma_{\theta EF}$) was derived. This parameter represents the directional spreading of the whole directional wave climate around the mean wave energy flux direction (θ_{EF}). It was considered as the directional proxy of the variability of wave directionality. The directional domain was divided into 360 directional bins with a high resolution of (1˚). The cumulative energy flux, in the whole time series, was calculated for each directional sector (1˚). Consequently, the probability of the energy flux of each directional bin (P_j) can be obtained as:

$$P_j=\frac{EF_j}{EF_{total}} \tag{4}$$

Where EF_j is the total wave energy flux of that directional sector within the wave climate and EF_{total} is the total cumulative wave energy flux of the time series of the entire wave climate where:

$$\sum_{j=0}^{j=2\pi} P_j = 1 \tag{5}$$

Finally, the one-sided directional width of the wave climate’s total energy flux distribution over the directions ($\sigma_{\theta EF}$), i.e. the directional spreading can be obtained as:

$$\sigma_{\theta EF}{}^{2} = \int_{-\pi}^{\pi} (\theta_j - \theta_{EF})^{2} * P_j = \int_{-\pi}^{\pi} \left[2\sin\left(\frac{\theta_j - \theta_{EF}}{2}\right)\right]^{2} * P_j \tag{6}$$

Where (θ_j) represents each directional sector with (θ_j = 1, 2, 3,….. 2π).

3.4. *Equilibrium planform of beaches*

In this regard, the SMC model was used in order to plot and best fit the static equilibrium planform for the selected beaches onto aerial vertical images. Wave climate parameters (e.g. the mean energy flux direction θ_{EF} and the wave period T_{p12}) in conjunction with the water depth (d) at the diffraction point were used in this procedure. The Parabolic Bay Shape Equation (PBSE) proposed by Hsu and Evans (1989) was utilized in this study in conjunction with the modification proposed by Tan and Chiew (1994) of applying the tangential boundary condition at the straight down-drift part of the bay beach. The plot of the best fit SEP was obtained selecting a free initial down-coast control point (P_o) for each beach defining the angle (α_{Po}). Furthermore, the planform shape was also plotted using the same procedure, but applying the (α_{min}) approach of González and Medina (2001). Figures 3 and 4 show the plot of both planforms and their corresponding angles for the Prinho and Cassino beaches in Brazil.

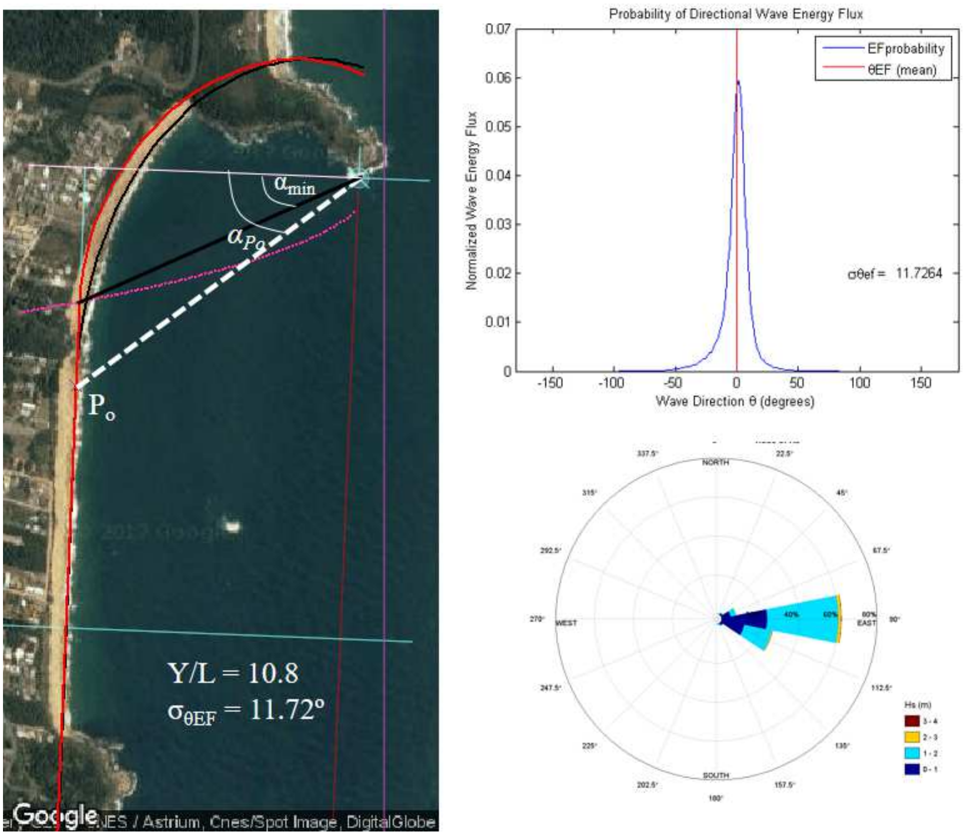

Figure 3. The best fit SEP (black line) and the planform shape using the approach proposed by González and Medina (2001) to locate the down-coast control point (P_o) via the (α_{min}) angle (red line) for the case of Prinho beach, Brazil. Wave rose and the directional distribution of the probability of wave energy flux are plotted for a wave climate time series of more than 60 years (from 1948 onwards) using the DOW database. Photos are based on Google Earth imagery

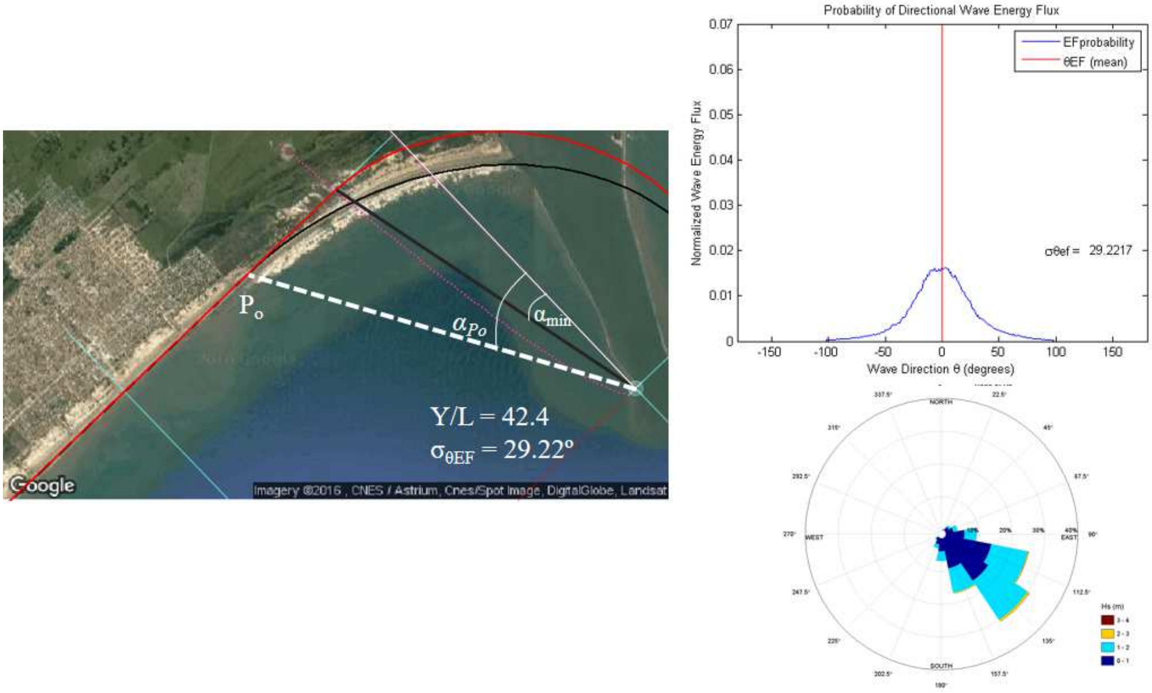

Figure 4. The best fit SEP (black line) and the planform shape using the approach proposed by González and Medina (2001) to locate the down-coast control point (P$_o$) via the (α_{min}) angle (red line) for the case of Cassino beach, Brazil. Wave rose and the directional distribution of the probability of wave energy flux are plotted for a wave climate time series of more than 60 years (from 1948 onwards) using the DOW database. Photos are based on Google Earth imagery

4. Results

The comparison between the (α_{min}) and (α_{Po}) angles to locate the downdrift point (P$_o$) and thus the (SEP) showed deviations between them for most of the cases, as seen in Figure 5. This can be observed in Figures 3 and 4 for two embayed beaches in Brazil. The beach at Prinho exemplifies a case with a diffraction point close to the coast with (Y/L> 10) and a considerable degree of wave climate directional variance. Moreover, the beach at Cassino represents a case in which the diffracting point is far from the shoreline with a wider directional wave variability. From the two plots it can be seen that the (α_{min}) approach for locating the down-coast control point (P$_o$) is no longer valid in such cases and that the down-coast control point has moved farther along the relatively straight segment of the bay. Furthermore, the (α_{Po}) angle based on the best fit (SEP) was plotted against the dimensionless distance (Y/L) for different degrees of wave climate directional variance, as seen in Figure 6, in addition to the plot of the (α_{min}) curve.

The results of the study have indicated that the formula of the (α_{min}) angle to locate the down-coast control point (P$_o$) of the parabolic bay shape is not valid for many cases as seen in Figure 6 that for the same (Y/L) value there is not a unique value for the (α_{Po}) angle. This clarified that the location of the (P$_o$) point, and thus the value of the (α_{Po}) angle, are not only dependent on (Y/L) but also governed by the directional variance of the nearshore wave climate, i.e. (α_{Po} = f (Y/L, $\sigma_{\theta EF}$)). A clear trend can be noticed that the wider the directional standard deviation ($\sigma_{\theta EF}$) of the wave climate near the diffraction point, the larger the (α_{Po}) angle and the farther the (P$_o$) point along the shoreline. Moreover, the farther the diffraction point from the shoreline, the smaller the affected part of the beach by the headland structure. The obtained results have proved the hypothesis that when the wave climate is characterized with a clear narrow directional sector, i.e. small ($\sigma_{\theta EF}$) values, the (α_{min}) formula is appropriate for only closer diffraction points (Y/L <10). Otherwise, when the diffraction point of the breakwater tip is far from the shoreline (Y/L >10) and/or the directional wave climate is wide banded the formula is no longer valid.

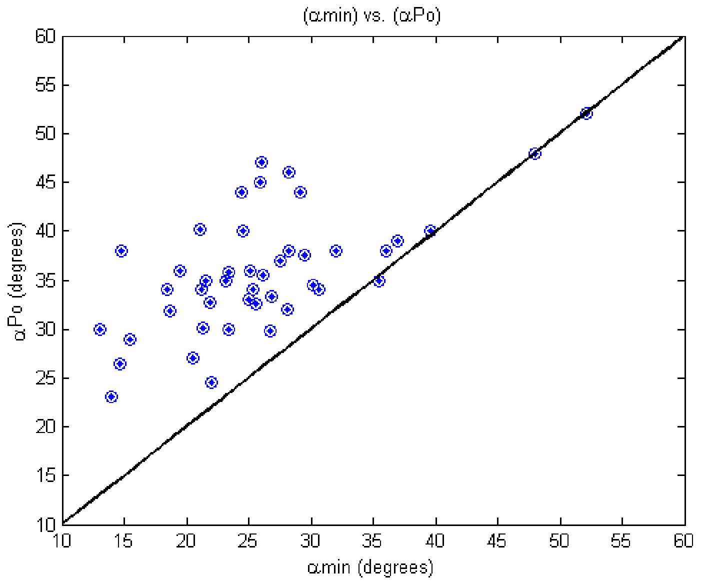

Figure 5. Comparison between (α_{min}) and (α_{Po}) best fit angles

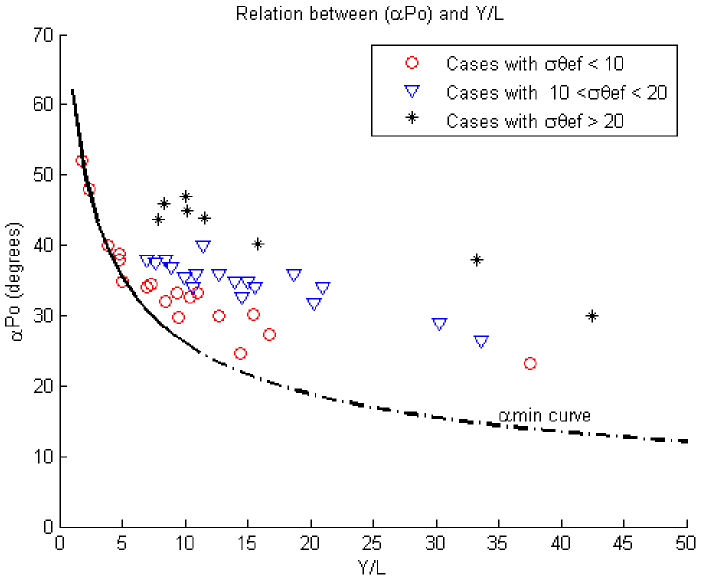

Figure 6. Relation between the (α_{Po}) best fit angle and (Y/L) for different degrees of wave climate directional variance

5. Discussion

When waves encounter an obstacle such as a protruding headland or a breakwater during propagation, refraction and diffraction are present in the lee of the structure. The dominant phenomenon is the lateral spreading of wave energy in the sheltered area which is caused by diffraction. This process plays an important role in sculpting the planform of embayed beaches. It is very sensitive to the directional spreading of waves approaching the breakwater. Goda (1985) stated that the degree of directional spreading of wave energy greatly affects the extent of wave refraction and diffraction. This indicates that wave directionality is very relevant to the determination of the sheltered area behind a coastal barrier and defines the part of the beach affected by the structure. Briggs et al. (1995), concluded that the directional spreading of waves is the most important factor governing the extent of wave diffraction in the lee of a breakwater and that it should be considered in diffraction analysis of engineering problems. Therefore, in order to properly define the static equilibrium planform of an embayed beach, the directional spreading of waves is very important and has to be incorporated in the analysis.

Accordingly, the behavior of prototype bayed beaches of the current study can be understood, considering the effect and importance of wave directionality. The results clearly indicate that the wider the directional spreading around the mean direction of energy flux, the larger the affected part of the beach and the farther the down-coast control point (P_o), which represents the end point of the curved part of the beach planform. This can be interpreted as the broader the directional spreading, the wider the extent of wave diffraction behind the headland, and consequently, the larger the (α_{Po}) angle which defines the location of (P_o). This also confirms that the (α_{min}) approach underestimates the location of (P_o) for beaches exposed to waves with high variability of wave directionality, and embayed beaches with diffraction points far from the shoreline. Diffracting points far from the coast means wider directional spreading conditions in most cases, in comparison with close shallower control points where refraction plays an important role in narrowing and decreasing the directional spreading.

It is worth highlighting that the calculation of the (α_{min}) angle and the corresponding location of the (P_o) point was derived by González and Medina (2001) based on the principle of monochromatic wave diffraction, ignoring the role of the directional spreading of the wave climate. Hence, that approach is valid only for structures close to the shoreline ($Y/L < 10$) with waves arriving from a dominant directional sector. Thus, it is valid for very shallow waters as waves arrive at the diffraction point from a narrow fan of directions due to refraction behaving like monochromatic waves. However, when the beach is exposed to a high variable modal wave climate and/or the diffraction point is far from the coast, the wave directional spreading plays a significant role in the diffraction process and should be considered using the directional random wave’s diffraction concept which is quite different from that of monochromatic waves. In this regard, the diffraction diagrams of directionally irregular waves given by Goda et al. (1978) and the best fit formulations for these curves introduced by Kraus (1984) are very useful in understanding the influence of directional spreading on the extent of wave diffraction. The diffraction coefficient (K_d) is a function of the parameter S_{max} which is the peak value of the (S) spreading parameter that defines the degree of concentration of the directional spreading. The wider the directional spreading of waves, the lower the spreading parameters (S and S_{max}) and the wider the directional width (σ_θ). Hence the diffraction coefficient is a function of the directional width of the wave energy directional distribution around the mean direction ($K_d = \mathrm{f}\ (\sigma_\theta)$).

According to (González and Medina, 2001), the point (P_o), represents the end of the section of the beach affected by the breakwater, i.e. the end of the diffraction extent, thus no effect of the structure, which yields to the condition that at P_o the coefficient K_d is ≥ 1. Consequently, the location of the (P_o) point is a function of the wave directional spreading (σ_θ). This explains the different behaviors of the obtained results for the prototype embayed beach cases. Consequently, it can be concluded that the implementation of the variability of directional wave climate is relevant and critical in defining the location of (P_o) and therefore the SEP and the curvature of headland bay beaches.

6. Conclusion

Headland bay beaches represent one of the most common physiographic features along the world’s coasts. The Static Equilibrium Planform (SEP) of these pocket beaches can be studied by applying the Parabolic Bay Shape Equation (PBSE) proposed by Hsu and Evans (1989). Employing 44 bay beaches in Spain and Latin America and exploiting the available long time series data of wave climates, the equilibrium shape in planform was studied using the PBSE with the modified down-coast alignment proposed by Tan and Chiew (1994). The directional wave climate close to the diffraction point of each embayed beach was analyzed defining the direction of the mean wave energy flux (θ_{EF}) and the directional spreading ($\sigma_{\theta EF}$) around it. The best fit planform of beaches indicated that the location of the down-coast control point (P_o), defined by the (α_{Po}) angle, from which the parabolic shoreline of the PBSE is valid, is not only a function of the offshore distance of the diffraction point from the straight segment of the shoreline (Y/L), but also the directional variance of the wave climate ($\sigma_{\theta EF}$). It was found that for the same (Y/L) distance, the wider the directional spreading of waves, the farther the location of the point (P_o). Furthermore, for the same degree of wave directional spreading, the farther the diffraction point from the shoreline, the smaller the part of the beach affected by the coastal barrier. The study has confirmed the hypothesis that the (α_{min}) approach underestimates the location of (P_o) for beaches exposed to waves with high variability of wave directionality, and bay beaches with diffraction points far from the shoreline. Consequently, the implementation of the directional variability of the wave climate is critical in defining the location of (P_o) and therefore the SEP of embayed beaches.

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